

Simulation of Gear Dynamics by Circuit Theory Methods

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Abstract

A methodology of study of gear dynamics with the aid of the circuit theory and linear graph methods is presented. In terms of analogy of the force, electric tension is used for the composition of equivalent electrical circuit, which immediately gives the equations of motion. The application of the electrical analogy method for the automation of composition of equation of motion and their analysis are considered for the classical example of a one-stage gear transmission with flexible supports and coupling masses. This technique can be extended to analyse the dynamic characteristics of more complex dynamic systems as a planetary transmission with flexible supports.

Index terms— dynamic gear model, equivalent electric circuit, linear graph, node equations, automation of composition of motion equations.

1 Introduction

The classical method of Lagrange equation (Genkin and Grinkevich, 1961) and the methods of dynamic stiffness or admittance (Airapetov et al. 1975) are used in studies of gear dynamics. These methods are cumbersome and laborious. In order to automate composition of equations of motion, the bond graph (Karnopp and Rosenberg, 1972) and some other methods are also used. However, the use of these methods presupposes presentation of a dynamic model as a system with concentrated parameters. Also, the bond graph of a relatively simple model, as, for example, a planetary gear transmission, is very cumbersome (Allen, 1979). On the other hand, electrical analogy method (Skudrzyk, 1968) and linear graph method (Mason and Zimmermann, 1960) allow us to model dynamic systems with both concentrated and distributed parameters (Podzharov, 1983, 1987, Sasa et al., 2004, Wojnarowski, 2006, Kalous, 2009).

In this paper a methodology of study of gear dynamics with the aid of the electric circuit theory methods is presented. The analogy between the force and electric tension is used to compose equivalent electric circuits for dynamic gear systems. Nomenclature: J -moment of inertia of the mass m , k -torsional stiffness $\frac{1}{2}$, S C -support stiffnesses, C -stiffness of tooth engagement, τ , τ_t -variable torsion moments, μ -moment

of inertia reduced to the mass moving on the line of action, C -torsion stiffness reduced to linear stiffness on the line of action, r -base radius of a gear, F -torsional moment reduced to a force applied in the line of action of gear engagement, δ -kinematic error in the gear engagement, y -element of the matrix of mechanical conductance, Y -matrix of mechanical conductance, Z -mechanical impedance, T -transmission in a graph between the points i and j , f -frequency, $\omega = 2\pi f$.

2 II.

3 Modelling One-Stage Gear Transmission

We shall now consider the use of electrical analogy for the automation of composition of equations of motion and their analysis in the example of one-stage gear transmission with flexible supports and coupled masses. The mechanical model of the transmission is shown in Fig. 1 and in Fig. 2, presenting the equivalent electrical

3 MODELLING ONE-STAGE GEAR TRANSMISSION

circuit. Here, the parameters of the torsion system reduced to the parameters of a linear system are determined as follows: $2 / b i i r J = \mu$, $2 / b i i k i r k C =$, $b i i r t T t F /) () (= (1)$

This system has 10 independent elements and 11 resonances and antiresonances (Skudrzyk, 1968), including zero and infinite frequencies. When the circuit is excited by variable tensions (forces) Let us consider stationary vibrations and assume that the gear is a linear dynamic system. Then, the solution of this system with periodic force or kinematic excitation can be found as a sum of harmonics. In this case the equations of motion can be composed as Kirchhoff equations of the equivalent electric circuit.) $(1 t F$ and $) (2 t F$ the contours

The total conductance between the points a and b will be equal to the sum of the conductances of parallel branches (Skudrzyk, 1968)., $4 3 2 1 0 Y Y Y Y Y Y Y Y Y Y a b + + + + = (2)$ where, $/ 2 0 C j Y ? = ()$, $/ 1 2 2 2 ? + = ? ? j C m j Y S 1 2 1 1 1 1 1 ? ? ? ? ? ? ? ? + ? ? ? ? ? ? ? ? + = \mu ? ? \mu ? j C j j Y$, $()$, $/ 1 3 3 3 ? + = ? ? j C m j Y S 1 3 1 3 4 4 1 ? ? ? ? ? ? ? ? + ? ? ? ? ? ? ? ? + = \mu ? ? \mu ? j C j j Y$. (3)

The input impedance between points a and b can be found as inverse of $a b Y a b a b Y Z / 1 =$. (4)

Substituting the equations (3) in the equation (??) and (4) and making $a b Z$ equal zero, we can find a frequency characteristic of $a b Z$, which is shown in Fig. 3 for the transmission with the following parameters: It was calculated neglecting the damping in the system and, according to the Foster theorem (Skudrzyk, 1968), it has a monotonous character. We can find from the curve that the poles are at frequencies 55 Hz, 209 Hz, 300 Hz, 794 Hz and 5200 Hz. The zeros are at the frequencies 115 Hz, 259.5 Hz, 408 Hz and 859.5 Hz.

The poles $\pi i f$ and zeros $s j f$ can also be found approximately from the concepts of parallel and successive resonances or resonances of tensions and currents: $H z C f p 50 2 1 4 1 1 = \mu ?$, (5) $H z m C C f S p 232 2 1 3 3 2 2 2 = + + = \mu ?$, $H z m C f S p 336 2 1 2 1 2 1 3 = + + = \mu \mu ?$, (7) $H z m C C f S p 778 2 1 2 23 2 1 4 = + + = \mu ?$, (8)

$() H z m m C f p 5162 2 1 1 3 1 2 1 3 1 2 2 5 = + + + = ? ? ? ? \mu \mu ?$ (9) $H z C f S 104 2 1 3 3 1 = \mu ?$, (10) $H z m C f S 5 . 259 2 1 3 3 2 = ?$, (11) $H z C f S 397 2 1 2 1 3 = \mu ?$, (12) $H z m C f S 5 . 859 2 1 2 2 4 = ?$, (13)

As we see, the approximate and exact frequencies are very similar to each other. Therefore, the formulas (5) -(13) can be used for the analysis of resonances.

Now, let us use the linear graph method (Mason and Zimmermann, 1960) to obtain a general form for this model. A linear graph for the circuit in Fig. ?? is constructed in Fig. ??. This graph illustrates the relations between forces $i F$ (upper nodes) and velocities $i v$

(lower nodes). The lines between them are transmissions, which in this case are mechanical admittances $i y$ or mechanical impedances $i z$. Thus, in this graph we use relations $i i i F y v ? =$ and $i i i v z F ? =$ (14)

In order to simplify the graph, the number of nodes can be reduced retaining only the nodes which we need to determine. Further simplification of the graph can be implemented by adding parallel transmissions and excluding the nodes, which we do not need to determine, by splitting them. Hence, splitting the nodes $i F \mu$, $C i v$, $i v \mu$ and excluding the loops $i l$, we can obtain the transformed graph presented in Fig. ??.

Here,) $(1 i i j y \mu ? \mu =$, $i C i C j y / ? =$, $) / (1 ? \mu ? j C j y i i S i + =$. (15)

The transmissions of this graph can be determined by the following formulas (16)

Where $1 2 1 1 /) (C y y l \mu \mu + ? =$, $3 4 3 3 /) (C y y l \mu \mu + ? =$, $2 3 2 3 2 2 /) (C S S y y y y l + + + ? = \mu \mu$. (17)

This graph can also be described by the equations determining the nodes in relation to adjacent nodes and transmissions that link them:) $() () (4 33 3 2 32 2 22 3 23 2 1 21 1 11 2 12 1 t F T F F T t S T F T F F T t F T F T F C C C C C C C = + ? = ? + ? = ? ?$ (18)

Substituting (16) and (17) in (??8) and multiplying each of the itch equation (??8 $t F F y F y t S F y F y F y t F y F y F y C C C C C C C = + ? = ? + ? = ? ? \mu$ (19) $P F Y c = \times$, (21)

Here, the matrix of the mechanical conductance Y is symmetrical; each itch diagonal element is positive and equal to the sum of the input mechanical conductances of the elements, which enter the itch node. Each non-diagonal element is negative and equal to the transition conductance of the elements, which locate between itch and j -th nodes. This type of matrix is known in the circuit theory as matrix of node equations (Karni, 1966). The excitation term in the right-hand part of the i -th equation is equal to the velocity of cinematic error in the gear engagement, in the case of cinematic excitation. In the case of force excitation, it equals to the product of the exciting force and the transition conductance between the point of application of the force and the itch node. Therefore, it is not necessary to compose equivalent electric circuits and graphs. Instead, following the rules explained above, we can directly compose the matrix of mechanical conductance and the vector of the right-hand part of the equations.

The above formulated rules can be extended to more complicated systems and systems with distributed parameters.

A dynamic calculation of the gear with the parameters mentioned above was implemented using the equations (15) -(20). The damping in elastic elements was considered by introducing complex stiffness (Skudrzyk, 1968).) $1 (i i i j C C ? + =$, (22)

Where $i ?$ -loss factor in i -th elastic element. 5) -(??). The measurement of noise and vibration of this gear shows that it has high levels at frequency 5200 Hz.

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Figure 1:

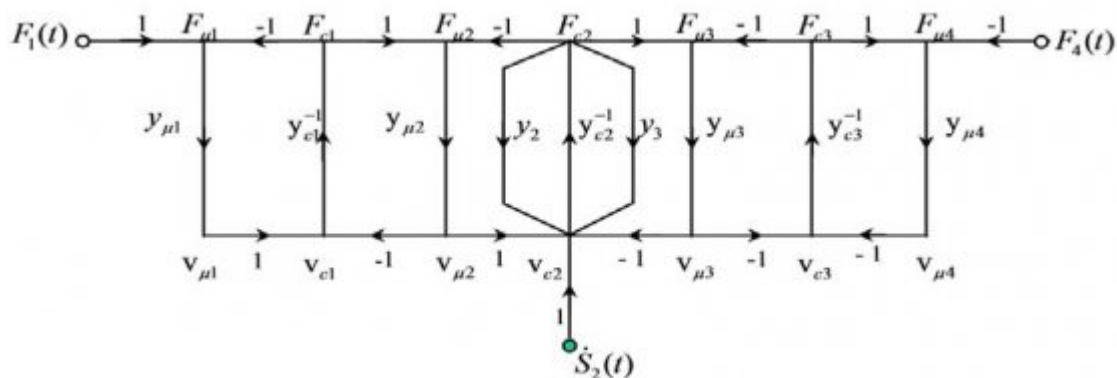


Figure 2: Where

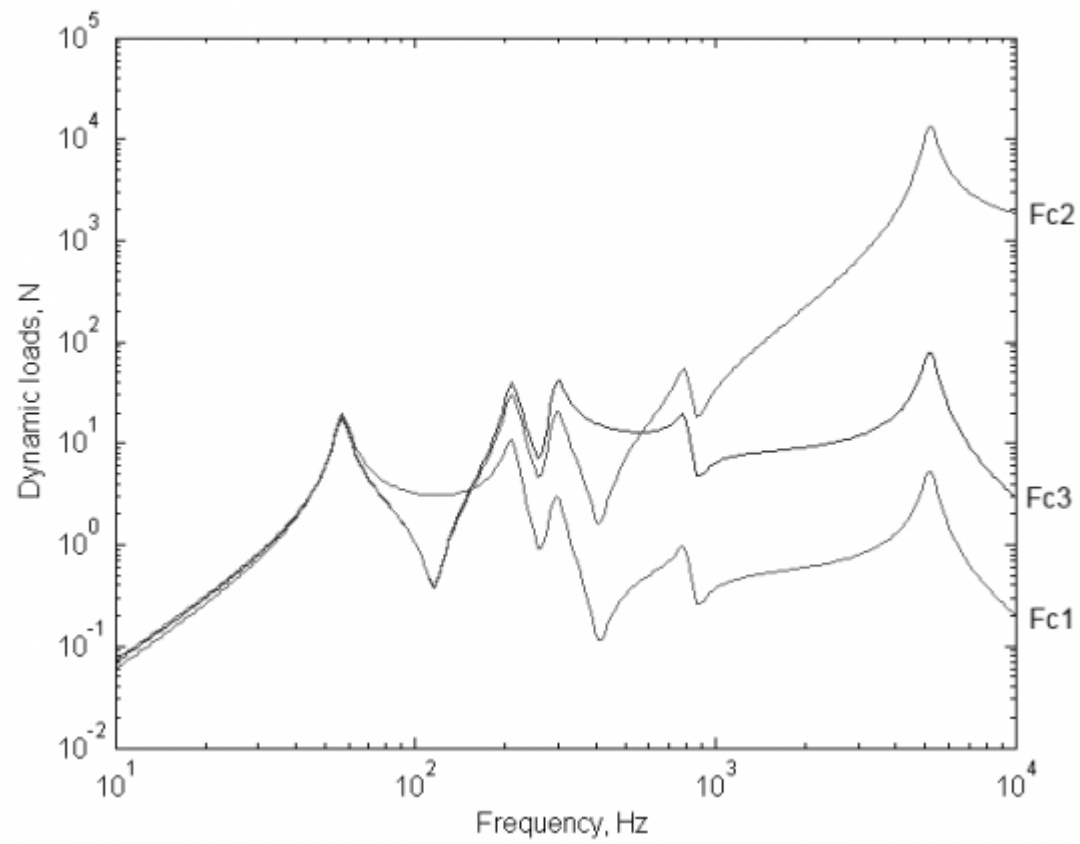


Figure 3:

1

| No. | Mechanical System | Electrical System |
|-----|-------------------|-----------------------|
| 1 | Force | Tension |
| 2 | Speed | Electric current |
| 3 | Displacement | Electric charge |
| 4 | Mass | Inductance |
| 5 | Flexibility | Capacitance |
| 6 | Absorber | Electrical resistance |

Figure 4: Table 1 :

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