Reflection and Transmission of Elastic Waves at a Loosely Bonded Interface between an Elastic Solid and a Viscoelastic Porous Solid Saturated by Viscous Liquid

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Abstract

In the present paper, a problem on reflection and transmission of elastic waves at a loosely bonded interface between an elastic solid and a viscoelastic porous solid saturated by viscous liquid is studied. The study is carried out with the assumption that the interface behaves like a dislocation which preserves the continuity of stress and allows a finite amount of slip. The appropriate potential functions for reflected and transmitted waves satisfy the required boundary conditions at the interface. The relations between amplitude ratios of different reflected and refracted waves are obtained for incidence of P and SV waves. The amplitude ratios of various reflected and refracted waves are computed for a particular model. The effects of loosely boundary and viscoelasticity are observed on these amplitude ratios.

Index terms— reflection and transmission of elastic waves, reflected and refracted waves are computed for a particular model.

1 Introduction

he observed attenuation of the seismic wave in the earth helps in getting information regarding the composition and state of deep interior. This attenuation cannot be explained by assuming the earth to be an elastic solid. **??**iot (1956a) studied the propagation of the plane harmonic seismic waves in liquid saturated porous solids. Biot (1962) presented a unified treatment of the mechanics of deformation and acoustic propagation in porous media, where liquidsolid medium is treated as a complex physico-chemical system with resultant relaxation and viscoelastic properties. Deresiewicz (1960) and Deresiewicz and Rice (1962) studied the reflection at the plane tractionfree surface of non-dissipative and dissipative liquid saturated porous solids respectively. They considered the porous solid as perfectly elastic with no internal energy loss.

Viscoelasticity is an important property of many rocks in the crust, which is a major cause of seismic attenuation. In the presence of porosity, a viscoelastic solid permeated by pores and fractures and saturated with viscous fluid becomes a more realistic model for sedimentary or reservoir rocks. Biot (1956b) established Author ? ?: Department of Mathematics, Post Graduate Government College, Sector 11, Chandigarh, India. e-mail: bsingh@gc11.ac.in the equations for the deformation of a viscoelastic porous solid containing a viscous fluid under the most general assumptions of anisotropy. Sharma and Gogna (1991) studied the seismic wave propagation in a viscoelastic porous solid saturated by viscous liquid. **??**ashishth et al. (1991) investigated a problem on reflection and transmission of a plane periodic wave incident on the loosely bonded interface between an elastic solid and a liquid-filled porous solid with the assumption that the interface behaves like a dislocation which preserves the continuity of stress allowing a finite amount of slip. **??**ashishth and Gogna (1993) studied a problem of reflection and refraction of plane seismic waves incident on an interface of two loosely bonded half-spaces, an elastic solid half-space and a liquidsaturated porous solid half-space, which permits a finite amount of slip. Vashishth and Sharma (2008) discussed the wave propagation in a medium considered as a viscoelastic, anisotropic and porous solid frame such that its pores of anisotropic permeability are filled with a viscous fluid. Recently, Sharma (2012) studied the propagation of Rayleigh waves on the stress-free surface of a viscoelastic, porous solid saturated with viscous fluid.

In the present paper, a problem is considered on reflection and transmission of elastic waves on loosely bonded interface between an elastic solid and a viscoelastic porous solid saturated by viscous liquid. For incidence of P and SV waves, the amplitude ratios of various reflected and refracted waves are computed for a particular model. The effects of loosely boundary and viscoelasticity are shown graphically on these amplitude ratios.

2 II.

3 Basic Assumptions

Murty (1976) introduced a real bonding parameter to which numerical values can be assigned corresponding to a given degree of bonding between half-spaces and discussed the particular cases of ideally smooth and fully bonded interfaces corresponding to the values 0 and ? of the bonding parameter. He considered three basic assumptions. The first assumption is that the stresses are continuous T Global Journal of Researches in Engineering ( ) Volume XIV Issue III Version I across the interface. The second assumption is that the microscopic structure of the material at the interface is such that a finite amount of slip can take place at the interface when a periodic wave is propagating. The third assumption is that there exists a linear relation between slip and shear stress at the interface which implies that different degrees of bonding correspond to different values of the constant of proportionality. The principle behind the third assumption is that there must exist some relation between the local shearing stress and the 'slip' at the interface such that when the shearing stress is zero, the 'slip' is infinite implying that the interface behaves like an ideally smooth interface and when the 'slip' vanishes the interface behaves as a fully bonded interface. We may assume that shearing stress = K × slip at the interface of loosely bonded media so that the vanishing of K corresponds to an ideally smooth interface and an infinitely large value of K corresponds to a welded interface. The intermediate values of K represent a loosely bonded interface.

We assume a model having a viscous liquid layer between the elastic half-space and liquid-saturated porous viscoelastic solid half-space. Let H be the thickness of the layer and ? be the coefficient of viscosity and ? 0 H implying that the thickness of the layer is infinitely small. It is reasonable to assume that the shearing stress at the interface is given by( ) ( ) ( ) ( ) µ ? µ ? ? ? ? ? ? + + + ? ? + ? ? = + ? ? ? ? ? ? 2 2 2 2

.

.f u M u M w u w t , (**4**) ( ) ( ) ( ) ? ? ? ? ? ? ? ? ? + ? = + + ? ? ? ? ? ? ? ? 2 2 [. .] f w M u M w u mw t t ,(**5**)

Where, the vector ? is pore fluid viscosity, and ? is permeability. ? and M are the elastic coefficients related to the coefficient of fluid content ? , unjacketed compressibility ? and jacketed incompressibility by1 K ? ? = ? , ( ) ? ? ? = + ? 2 1/ M K .

The stresses ij ? and liquid pressure f p are given by solid ( )2 2 ij ij ij e M e M ? µ ? ? ? ? ? ? ? = + + + ? ? , ? ? = ? f e p M M ,(**6**)

Where ( ) , ,**12** ij i j j i e u u = +J xz u z ? ? ? ? ? = ? ? ? ? ? ? ,(**1**)

where u ? is the component of velocity parallel to the interface (dot represents time derivative) and the partial derivative is taken normal to the interface. Equation (1) can be approximated as( ) xz e u u H ? ? = ? ? ? ,(**2**)

where ( )e u u ? ? ?

is the jump in the x-component of velocity across the layer. If we assume the waves to be time harmonic, then equation (2) can be written as ( )xz e i u u H ? ? ? ? ? = ? ? ? ? ? ? , (**3**)

where ? is the angular frequency and u and e u are the displacement components parallel to the interface at the boundaries of the infinitesimal thin layer of viscous liquid.

III.

4 Basic Equations

According to Biot (1962), the differential equations governing the displacement u ? of solid matrix and U ? of interstitial liquid in a homogeneous isotropic porous solid saturated by viscous liquid are

Reflection and Transmission of Elastic Waves at a Loosely Bonded Interface between an Elastic Solid and a Viscoelastic Porous Solid Saturated by Viscous Liquid2 3 K ? µ ? ? = + ? ? ? ?

and, the stress-strain relations are ( )2 * * * * * * 2 ij ij ij e M e M ? µ ? ? ? ? ? ? ? = + + + ? ? , ? ? = ? ? * * * f p M e M ,(**8**)

With the help of (**??**), the equations of motion (4) and (5) become ( )( ) µ ? µ ? ? ? ? ? ? ? + + + ? + ? = + ? ? ? ? ? 2 2 * 2 * * * * * * 2 f u M e M u w t ,(**9**)( ) ( ) ? ? ? ? ? ? ? ? ? + = + + ? ? ? ? ? ? 2 * * * 2 f w M e M u mw t t , (**10**) ( ) ,0, u u w = ? , ( ) ,0, U W ? = ? , ( ) ,0, e e e u u w = ? ,(**12**)

where 12 11 u x x z ? ? ? ? ? ? = + + ? ? ? , 12 11 w z z x ? ? ? ? ? ? = + ? ? ? ? , ( ) ? ? = ? ? 1 , 12 11 1 2 0 U x x z ? ? ? µ µ ? ? ? ? = + + ? ? ? , 12 11 1 2 0 d W z z x ? ? ? µ µ ? ? ? = + ? ? ? ? , ,**e**? ? ? ? ? ? ? ? = + = ? ? ? ? ? ? , ( ) e e ? ? = ? ? , ()( ) * * * 2 * 2 / , 1,2 f j j f v j m i ? ? ? ? µ µ ? ? ? ?? ? + + = = ? ? ? + ? ? ? ? , ( ) ? ? ? ? ?? = ? + 0 . / f m i IV.

5 Reflection and Transmission

For incidence of P or SV wave, there will be reflected P, SV waves in elastic half-space and refracted * * 2 * 2 , 1,2 j j v j ? µ ? + = = * 2 3 * 3 v µ ? = . (**11**)

To consider only two-dimensional reflection problem, we shall restrict the plane wave solutions for the displacement potentials to those that have propagation and attenuation vectors in the x-z plane. Following Sharma and Gogna (1991), the components of displacement vectors are taken as

The appropriate potentials in elastic half-space are ( ) ( )0 1 i kx d z t i kx d z t e A e A e ? ? ? ? ? ? ? ? + ? = + , (**13**) ( ) ( ) * 0 2 i kx d z t i kx d z t e A e A e ? ? ? ? ? ? ? ? + ? = + , (**14**)

where1 2 2 2 . . d p v k ? ? ? ? ? ? ? ? ? ? = ? ? ? ? ? ? ? ? ? ? ? ? , 1 2 2 2 . . d p v k ? ? ? ? ? ? ? ? ? ? = ? ? ? ? ? ? ? ? ? ? ? ? .

The appropriate potentials in viscoelastic porous solid half-space are( ) ( ) ? ? ? ? ? ? ? = ? ? ? ? ? 12 12

11 11A r i P r t B e e ,**(15) ( ) ( ) ? ?**? ? ? ? = ? ? ? ? ? 22 22

12 21A r i P r t B e e ,**(16) ( ) ( ) ? ?**? ? ? ? = ? ? ? ? ? 32 32 12 A r i P r t C e e , (**17**)

where, the propagation vectors ij P ? and attenuation vectors ij A ? are defined by( ) Re Re ?1 j ij i P k x dv z = + ? ? , ( ) = + ? ? Im Im ?1 , j ij i A k x dv z with 1 2 2 2 2 i i dv p v k v ? ? ? = ? ? ? ? ? ? ? . where = ? ? 12 1 ( ) y C C , ? 1 C is arbitrary complex vector chosen such that ? ? ? = ? 0, k is an arbitrary complex number such that Re 0 k ? to ensure

propagation in the positive x-direction. The subscripts Re and Im denote the real and imaginary parts of the corresponding complex quantities. Following Borcherdt (1982), the displacement potentials given by (13) to (17) ? ? ? ? ? ? ? = = = = = = = ? ? ? * * *( ) ? ? = ? = ? * Im 2 sin ,**(1,2,3)**j j j k A j (19)

Which is the extension of Snell's law. We also obtain the following non-homogeneous system of five equations= = = ? 5 1 , (1,2,..,5) ij j i j a Z b i ,**(20)**

where ( )( ) ? ? µ ? µ ? = ? ? + = 2 2 11 12 2 , 2 a k d a kd , ( )( ) ( ) { } 2 2 * * * 2 * * * * * * 13 1 1 1 1 a H M dv k M M M ? ? ? ? µ µ = ? + + ? ? ? ? ( ) ( ) µ ? + ? ? 2 * * * 1 1 , M M dv ( )( ) ( ) 2 2 * * * 2 * * * * * * 14 2 2 2 1 a H M dv k M M M ? ? ? ? µ µ ? ? = ? + + ? ? ? ? ? ( )( ) µ ? + ? 2 * * * 2 2 , M M dv ( ) ? ? = ? ? 2 * * * * 15 3 , a kdv H M ( ) µ ? µ ? µ ? ? = ? = ? ? = ? ? ? 2 2 *µ ? ? µ = ? = ? ? ? ? ? ? ? ? ? = ? = ? ? ? 2 51 52 0 0 1 1 , 1 sin 1 sin a k a kd ? ? ? ? ? ? = ? ? = ? ? ? ? 2 2 53 1 54 2 0 0 1 1 2 , 2 , 1 sin 1 sin a kdv k a kdv k ( ) { } ? ? ? = ? ? ? ? 2 2 55 3 3 0 1 , 1 sin a kdv dv k (a)

For incident P wave, A Z A = , 2 2 0 A Z A = , 11 3 0 B Z A = , 21 4 0 B Z A = , 12 5 0 C Z A =

, Are amplitude ratios of reflected P, reflected SV, refracted P 12 , refracted P 22 and refracted P 32 waves, respectively. A Z A = , 2 2 * 0 A Z A = , 11 3 * 0 B Z A = , 21 4 * 0 B Z A = , 12 5 * 0 C Z A =

, Are amplitude ratios of reflected P, reflected SV, refracted P 12 , refracted P 22 and refracted P 32 waves, respectively. For ? = 1 , the above system of equations (20) reduces for welded interface. , , , Z Z Z Z and 5 Z , given by (20), are computed for incident P and SV waves. The angle of incidence 0 ? , is considered to be varying from normal incidence ( ) 0 0 ? = ° to grazing incidence ( )0 90 ? = °.

We restrict the numerical computations for homogeneous case only. a) Loosely Boundary Effect i. Incident P wave The amplitude ratios of reflected P and SV waves for ? = 0.25, 0.5, 0.75 and 1.0 are plotted against the angle of incidence (0 o < ? 0 < 90 o) of P wave. These variations are shown in Figures 2 and 3 by black, blue, red and green curves, respectively. In each case, the amplitude ratios of reflected P and SV waves are same at normal and grazing incidence. The comparison of the different curves shows the effect of loose boundary on amplitude ratios of reflected P and SV waves. This effect is observed maximum in the range 45 o < ? 0 < 90 o . The amplitude ratios of refracted P 12 , P 22 and P 32 waves for ? = 0.25, 0.5, 0.75 and 1.0 are plotted against the angle of incidence of P wave. These variations are shown in Figures 4 to 6 by black, blue, red and green curves, respectively. These amplitude ratios are also affected due to loosely boundary at angles other than grazing and normal incidence.

ii. Incident SV wave

The amplitude ratios of reflected P, SV waves and refracted P 12 , P 22 and P 32 waves for ? = 0.25, 0.5, 0.75 and 1.0 are plotted against the angle of incidence (0 < ? 0 < 50) of SV wave also. These variations are J V.

6 Numerical Results and Discussion

For numerical computations of reflection and transmission coefficients, we resolve the operators t c c f f f f < < =

. Following Murphy III (1982), we consider water-saturated Massilon-sandstone with the following parameters: Porosity = 23 per cent, Grain density = 2.66 gm/cm 3, Pore diameter = 3 × 10 -3 cm. Following Biot (1956b), in case of water in the pores at 15°C, we find where, for uniform circular pores with axes parallel to the pressure gradient, c would be equal to 1.

Following Fatt (1959) and Yew and Jogi (1976) relevant elastic parameters for water-saturated sandstone are chosen to be shown in Figures 7 to 11 by black, blue, red and green curves, respectively. The comparison of the different curves shows the effect of loosely boundary on amplitude ratios of reflected and refracted waves.

7 b) Viscoelastic effect

To observe the viscoelastic effect on reflected and transmitted coefficients, we consider the incidence of P wave and ? = 0.25. On comparing the solid and dotted curves in Figures 12 to 16, it can be seen that the coefficients of reflected and transmitted waves change due to viscoelastic effect.

8 VI.

9 Concluding Remarks

Relations between reflection and transmission coefficients are obtained for incident of P and SV at a loosely bonded interface between an elastic solid halfspace and a viscoelastic porous solid half-space. Numerical values of these coefficients are computed for a particular model of the interface. It is observed that these coefficients are affected significantly due to the presence of loosely boundary. These coefficients are also affected due to the presence of viscoelasticity in upper half-space. Figure **??**4 : Viscoelastic effect on the amplitude ratios of transmitted P 12 wave against the angle of incidence of P wave for ? = 0.25

Figure **??**5 : Viscoelastic effect on the amplitude ratios of transmitted P 22 wave against the angle of incidence of P wave for ? = 0.25 [1] [2]

Figure 1:

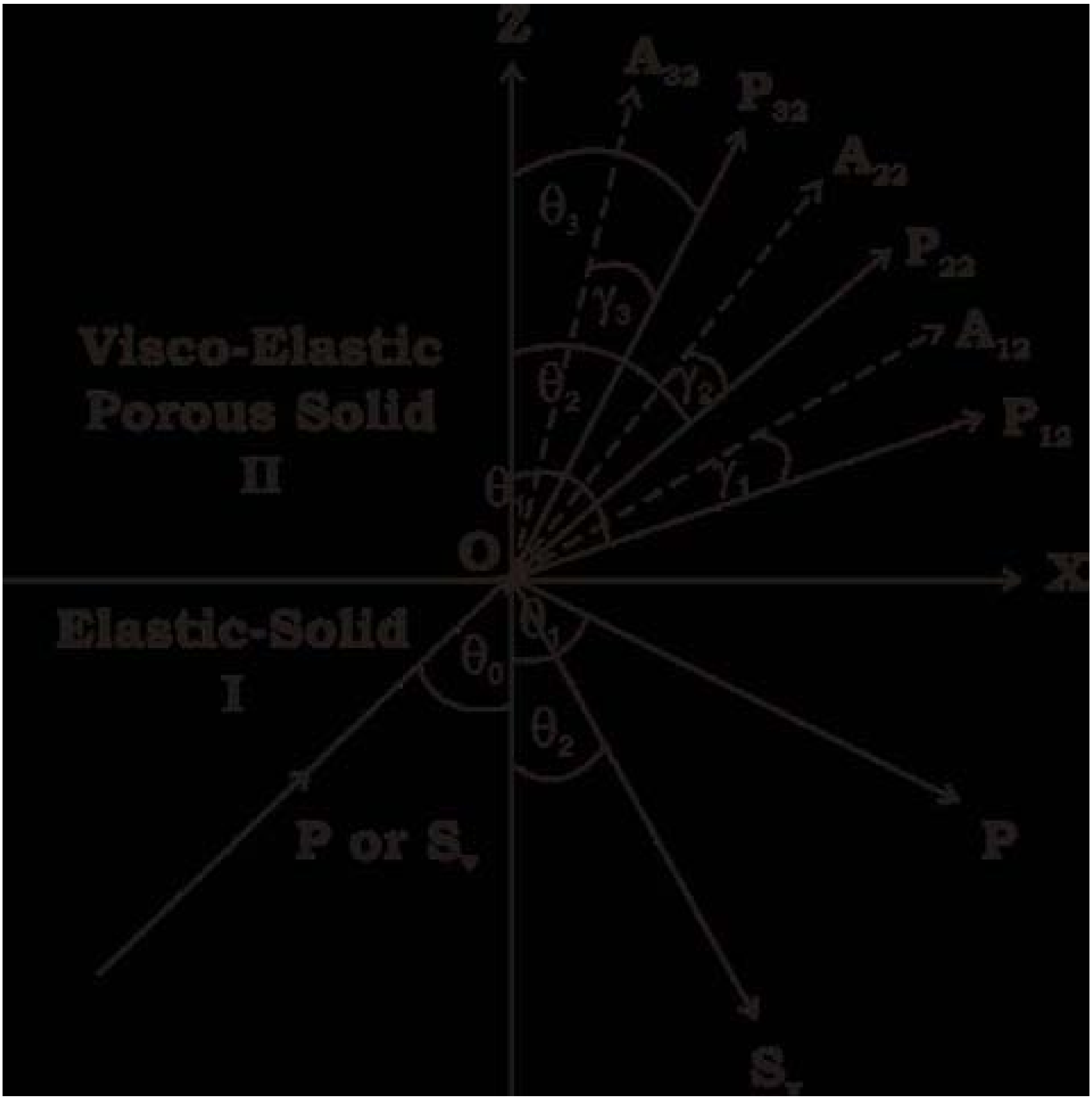
Z
A32
P32
A22
P22
A12
P12
Visco-Elastic
Porous Solid
II
O
X
Elastic-Solid
I
P or Sv
P
Sv

Figure 2:

[Silva ()] 'Body Waves in Layered Anelastic Solid'. W Silva . *Bull. Seismol Soc. Am* 1976. 66 p. .

[Murphy ()] 'Effects of Partial Water Saturation in Massilon-sandstone and Vycor Porous Glass'. Iii Murphy , WF . *J. Acoust. Soc. Am* 1982. 71 p. .

[Biot ()] 'Mechanics of Deformation and Acoustic Propagation in Porous Media'. M A Biot . *J. Appl. Phys* 1962. 33 p. .

[Vashishth and Sharma ()] 'Propagation of plane waves in poroviscoelastic anisotropic media'. A K Vashishth , M D Sharma . *Applied Mathematics and Mechanics* 2008. 29 p. .

[Sharma ()] 'Rayleigh waves in dissipative poro-viscoelastic media'. M D Sharma . *Bull. Seismo. Soc. Am* 2012. 102 p. .

[Vashisth et al. ()] 'Reflection and transmission of elastic waves at a loosely bonded interface between an elastic solid and liquid-saturated porous solid'. A K Vashisth , M D Sharma , M L Gogna . *Geophy. J. Int* 1991. 105 p. .

[Murty ()] 'Reflection, transmission and attenuation of elastic waves at a loosely-bonded interface of two half spaces'. G S Murty . *Geophys. J. R. astr. Soc* 1976. 44 p. .

[Borcherdt ()] 'Reflection-refraction of General P and Type-I S Waves in Elastic and Anelastic Solids'. R D Borcherdt . *Geophys. J. R. Astr. Soc* 1982. 70 p. .

[Sharma and Gogna ()] 'Seismic wave propagation in a viscoelastic porous solid saturated by viscous liquid'. M D Sharma , M L Gogna . *Pure and Applied Geophysics (PAGEOPH)* 1991. 135 p. .

[Yew and Jogi ()] 'Study of Wave Motions in Fluid-saturated Porous Rocks'. C H Yew , P N Jogi . *Acoust. Soc. Am* 1976. 60 p. .

[Fatt ()] 'The Biot-Willis Elastic Coefficients for Sandstone'. I Fatt . *J. Appl. Mech* 1959. 26 p. .

[Deresiewicz ()] 'The effect of boundaries on wave propagation in a liquid-filled porous solid I. Reflection of plane waves at a free plane boundary (non-dissipative case)'. H Deresiewicz . *Bull. Seismo. Soc. Am* 1960. 50 p. .

[Deresiewicz and Rice ()] 'The effect of boundaries on wave propagation in a liquid-filled porous solid: III. Reflection of plane waves at a free plane boundary (general case)'. H Deresiewicz , J T Rice . *Bull. Seismo. Soc. Am* 1962. 52 p. .

[Vashisth and Gogna ()] 'The effect of loose boundaries on wave propagation in a porous solid: Reflection and refraction of seismic waves across a plane interface'. A K Vashisth , M L Gogna . *Int. J. Solid Struct* 1993. 30 p. .

[MA ()] 'The Theory of Propagation of Elastic Waves in Fluid-saturated Porous Solids'. MA . *J. Acoust, Soc. Am* 1956a. 28 p. .

[Biot ()] 'Theory of deformation of a porous viscoelastic anisotropic solid'. M A Biot . *J. Appl. Phys* 1956b. 27 p. 459.

[Zwikker and Kosten ()] C Zwikker , C W Kosten . *Sound Absorbing Materials*, (New York) 1949. Elsevier Publishing Company, Inc.