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Reflection and Transmission of Elastic Waves at a Loosely Bonded Interface between an Elastic Solid and a Viscoelastic Porous Solid Saturated by Viscous Liquid

Baljeet Singh^α & Surjeet Singh^σ

Abstract- In the present paper, a problem on reflection and transmission of elastic waves at a loosely bonded interface between an elastic solid and a viscoelastic porous solid saturated by viscous liquid is studied. The study is carried out with the assumption that the interface behaves like a dislocation which preserves the continuity of stress and allows a finite amount of slip. The appropriate potential functions for reflected and transmitted waves satisfy the required boundary conditions at the interface. The relations between amplitude ratios of different reflected and refracted waves are obtained for incidence of P and SV waves. The amplitude ratios of various reflected and refracted waves are computed for a particular model. The effects of loosely boundary and viscoelasticity are observed on these amplitude ratios.

I. INTRODUCTION

The observed attenuation of the seismic wave in the earth helps in getting information regarding the composition and state of deep interior. This attenuation cannot be explained by assuming the earth to be an elastic solid. Biot (1956a) studied the propagation of the plane harmonic seismic waves in liquid saturated porous solids. Biot (1962) presented a unified treatment of the mechanics of deformation and acoustic propagation in porous media, where liquid-solid medium is treated as a complex physico-chemical system with resultant relaxation and viscoelastic properties. Deresiewicz (1960) and Deresiewicz and Rice (1962) studied the reflection at the plane traction-free surface of non-dissipative and dissipative liquid saturated porous solids respectively. They considered the porous solid as perfectly elastic with no internal energy loss.

Viscoelasticity is an important property of many rocks in the crust, which is a major cause of seismic attenuation. In the presence of porosity, a viscoelastic solid permeated by pores and fractures and saturated with viscous fluid becomes a more realistic model for sedimentary or reservoir rocks. Biot (1956b) established

the equations for the deformation of a viscoelastic porous solid containing a viscous fluid under the most general assumptions of anisotropy. Sharma and Gogna (1991) studied the seismic wave propagation in a viscoelastic porous solid saturated by viscous liquid. Vashishth et al. (1991) investigated a problem on reflection and transmission of a plane periodic wave incident on the loosely bonded interface between an elastic solid and a liquid-filled porous solid with the assumption that the interface behaves like a dislocation which preserves the continuity of stress allowing a finite amount of slip. Vashishth and Gogna (1993) studied a problem of reflection and refraction of plane seismic waves incident on an interface of two loosely bonded half-spaces, an elastic solid half-space and a liquid-saturated porous solid half-space, which permits a finite amount of slip. Vashishth and Sharma (2008) discussed the wave propagation in a medium considered as a viscoelastic, anisotropic and porous solid frame such that its pores of anisotropic permeability are filled with a viscous fluid. Recently, Sharma (2012) studied the propagation of Rayleigh waves on the stress-free surface of a viscoelastic, porous solid saturated with viscous fluid.

In the present paper, a problem is considered on reflection and transmission of elastic waves on loosely bonded interface between an elastic solid and a viscoelastic porous solid saturated by viscous liquid. For incidence of P and SV waves, the amplitude ratios of various reflected and refracted waves are computed for a particular model. The effects of loosely boundary and viscoelasticity are shown graphically on these amplitude ratios.

II. BASIC ASSUMPTIONS

Murty (1976) introduced a real bonding parameter to which numerical values can be assigned corresponding to a given degree of bonding between half-spaces and discussed the particular cases of ideally smooth and fully bonded interfaces corresponding to the values 0 and ∞ of the bonding parameter. He considered three basic assumptions. The first assumption is that the stresses are continuous

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across the interface. The second assumption is that the microscopic structure of the material at the interface is such that a finite amount of slip can take place at the interface when a periodic wave is propagating. The third assumption is that there exists a linear relation between slip and shear stress at the interface which implies that different degrees of bonding correspond to different values of the constant of proportionality. The principle behind the third assumption is that there must exist some relation between the local shearing stress and the 'slip' at the interface such that when the shearing stress is zero, the 'slip' is infinite implying that the interface behaves like an ideally smooth interface and when the 'slip' vanishes the interface behaves as a fully bonded interface. We may assume that shearing stress = $K \times$ slip at the interface of loosely bonded media so that the vanishing of K corresponds to an ideally smooth interface and an infinitely large value of K corresponds to a welded interface. The intermediate values of K represent a loosely bonded interface.

We assume a model having a viscous liquid layer between the elastic half-space and liquid-saturated porous viscoelastic solid half-space. Let H be the thickness of the layer and ξ be the coefficient of viscosity and $H \rightarrow 0$ implying that the thickness of the layer is infinitely small. It is reasonable to assume that the shearing stress at the interface is given by

$$\sigma_{xz} = \xi \left(\frac{\partial \dot{u}}{\partial z} \right), \quad (1)$$

where \dot{u} is the component of velocity parallel to the interface (dot represents time derivative) and the partial derivative is taken normal to the interface. Equation (1) can be approximated as

$$\sigma_{xz} = \frac{\xi}{H} (\dot{u} - \dot{u}_e), \quad (2)$$

where $(\dot{u} - \dot{u}_e)$ is the jump in the x -component of velocity across the layer. If we assume the waves to be time harmonic, then equation (2) can be written as

$$\sigma_{xz} = -i\omega \left(\frac{\xi}{H} \right) (u - u_e), \quad (3)$$

where ω is the angular frequency and u and u_e are the displacement components parallel to the interface at the boundaries of the infinitesimal thin layer of viscous liquid.

III. BASIC EQUATIONS

According to Biot (1962), the differential equations governing the displacement \bar{u} of solid matrix and \bar{w} of interstitial liquid in a homogeneous isotropic porous solid saturated by viscous liquid are

$$\mu \nabla^2 \bar{u} + (\lambda + \mu + \alpha^2 M) \nabla (\nabla \cdot \bar{u}) + \alpha M \nabla (\nabla \cdot \bar{w}) = \frac{\partial^2}{\partial t^2} (\rho \bar{u} + \rho_f \bar{w}), \quad (4)$$

$$\nabla [\alpha M (\nabla \cdot \bar{u}) + M (\nabla \cdot \bar{w})] = \frac{\partial^2}{\partial t^2} (\rho_f \bar{u} + m \bar{w}) + \frac{\eta}{\chi} \frac{\partial \bar{w}}{\partial t}, \quad (5)$$

Where, the vector $\bar{w} = \beta (\bar{U} - \bar{u})$ represents the flow of liquid relative to the solid measured in terms of volume per unit area of the bulk medium, λ, μ are Lamé's constants for the solid, ρ is mass density of the bulk material, ρ_f is mass density of liquid, m is Biot's parameter which depends upon porosity β and ρ_f , η is pore fluid viscosity, and χ is permeability. α and

M are the elastic coefficients related to the coefficient of fluid content γ , unjacketed compressibility δ and jacketed incompressibility $\kappa (= \lambda + \frac{2}{3} \mu)$ by $\alpha = 1 - \delta K$, $M = 1 / (\gamma + \delta - \delta^2 K)$.

The stresses τ_{ij} and liquid pressure p_f are given by solid

$$\tau_{ij} = 2\mu e_{ij} + \left[(\lambda + \alpha^2 M) e + \alpha M \xi \right] \delta_{ij}, \quad p_f = \alpha M_e - M \xi, \quad (6)$$

Where $e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})$ is strain tensor and $e = \text{div } \bar{u}$, $\xi = \text{div } \bar{w}$ are the dilatations. The viscoelastic and relaxation properties are obtained by

replacing the elastic coefficients λ, μ, γ and δ by the operators $\lambda^*, \mu^*, \gamma^*$ and δ^* respectively. Applying the correspondence principle, we have

$$K^* = \lambda^* + \frac{2}{3} \mu^*, \quad \alpha^* = 1 - \delta^* K^*, \quad M^* = 1 / (\gamma^* + \delta^* - \delta^{*2} K^*) \quad (7)$$

and, the stress-strain relations are

$$\tau_{ij} = 2\mu^* e_{ij} + \left[(\lambda^* + \alpha^{*2} M^*) e + \alpha^* M^* \xi \right] \delta_{ij}, \quad p_f = -\alpha^* M^* e - M^* \xi, \quad (8)$$

With the help of (7), the equations of motion (4) and (5) become

$$\mu^* \nabla^2 \bar{u} + (\lambda^* + \mu^* + \alpha^{*2} M^*) \nabla e + \alpha^* M^* \nabla \xi = \frac{\partial^2}{\partial t^2} (\rho \bar{u} + \rho_f \bar{w}), \quad (9)$$

$$\nabla (\alpha^* M^* e + M^* \xi) = \frac{\partial^2}{\partial t^2} (\rho_f \bar{u} + m \bar{w}) + \frac{\eta}{\chi} \frac{\partial \bar{w}}{\partial t}, \quad (10)$$

Following Sharma and Gogna (1991), in an unbounded viscoelastic porous solid saturated by viscous liquid, two dilatational waves of first and second kinds (P_1 wave and P_{11} wave) and one shear wave propagate. They also obtained the velocities v_j , ($j = 1, 2$) of dilatational waves and the velocity v_3 of shear wave as

$$v_j^2 = \frac{\lambda^* + 2\mu^*}{\rho_j^*}, \quad (j = 1, 2) \quad v_3^2 = \frac{\mu^*}{\rho_3^*}. \quad (11)$$

To consider only two-dimensional reflection problem, we shall restrict the plane wave solutions for the displacement potentials to those that have propagation and attenuation vectors in the x-z plane. Following Sharma and Gogna (1991), the components of displacement vectors are taken as

$$\bar{u} = (u, 0, w), \quad \bar{w} = (U, 0, W), \quad \bar{u}_e = (u_e, 0, w_e), \quad (12)$$

where

$$u = \frac{\partial \phi_{11}}{\partial x} + \frac{\partial \phi_{12}}{\partial x} + \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \phi_{11}}{\partial z} + \frac{\partial \phi_{12}}{\partial z} - \frac{\partial \psi}{\partial x}, \quad \psi = (-\bar{\psi}_1),$$

$$U = \mu_1 \frac{\partial \phi_{11}}{\partial x} + \mu_2 \frac{\partial \phi_{12}}{\partial x} + \alpha_0 \frac{\partial \psi}{\partial z}, \quad W = \mu_1 \frac{\partial \phi_{11}}{\partial z} + \mu_2 \frac{\partial \phi_{12}}{\partial z} - \alpha_0 \frac{\partial \psi}{\partial x},$$

$$u_e = \frac{\partial \phi_e}{\partial x} + \frac{\partial \psi_e}{\partial z}, \quad w_e = \frac{\partial \phi_e}{\partial z} - \frac{\partial \psi_e}{\partial x}, \quad \psi_e = (-\bar{\psi}_e),$$

$$\mu_j = \frac{\rho_f \alpha^* - \rho + (\lambda^* + 2\mu^*) / v_j^2}{\rho_f - \left(m + i \frac{\eta}{\omega \chi} \right) \alpha^*}, \quad (j = 1, 2), \quad \alpha_0 = -\frac{\rho_f}{(m + i\eta / \omega \chi)}.$$

IV. REFLECTION AND TRANSMISSION

For incidence of P or SV wave, there will be reflected P , SV waves in elastic half-space and refracted P_1 , P_{11} and SV waves in viscoelastic porous solid as shown in Figure 1.

The appropriate boundary conditions at a loosely bonded interface $z = 0$ between elastic solid and viscoelastic porous solid half-spaces are

i. $(\tau_{zz})_I = (\tau_{zz})_{II} + (p_f)_{II}$,

ii. $(\tau_{xz})_I = (\tau_{xz})_{II}$,

iii. $(w)_I = (w)_{II}$,

iv. $(\dot{w} - \dot{W})_{II} = (0)_I$,

v. $\tau_{xz} = \tau_0 (u - u_e)$, $\tau_0 = -ik\mu^* \frac{\psi}{1 - \psi \sin \theta_0}$.

The appropriate potentials in elastic half-space are

$$\phi_e = A_0 e^{i(kx - \alpha z - \omega t)} + A_1 e^{i(kx + \alpha z - \omega t)}, \quad (13)$$

$$\psi_e = A_0^* e^{i(kx - d\beta z - \omega t)} + A_2 e^{i(kx + d\beta z - \omega t)}, \quad (14)$$

$$\text{where } d\alpha = p.v. \left\{ \left(\frac{\omega}{\alpha} \right)^2 - k^2 \right\}^{\frac{1}{2}}, \quad d\beta = p.v. \left\{ \left(\frac{\omega}{\beta} \right)^2 - k^2 \right\}^{\frac{1}{2}}.$$

The appropriate potentials in viscoelastic porous solid half-space are

$$\phi_{11} = B_{11} e^{(-\bar{A}_{12} \cdot \bar{r})} \cdot e^{i(\bar{P}_{12} \cdot \bar{r} - \omega t)}, \quad (15)$$

$$\phi_{12} = B_{21} e^{(-\bar{A}_{22} \cdot \bar{r})} \cdot e^{i(\bar{P}_{22} \cdot \bar{r} - \omega t)}, \quad (16)$$

$$\psi = C_{12} e^{(-\bar{A}_{32} \cdot \bar{r})} \cdot e^{i(\bar{P}_{32} \cdot \bar{r} - \omega t)}, \quad (17)$$

where, the propagation vectors \bar{P}_{ij} and attenuation vectors \bar{A}_{ij} are defined by

$$\bar{P}_{ij} = k_{\text{Re}} \hat{x} + (-1)^j dv_{i_{\text{Re}}} \hat{z}, \quad \bar{A}_{ij} = k_{\text{Im}} \hat{x} + (-1)^j dv_{i_{\text{Im}}} \hat{z}, \quad \text{with } dv_i = p \cdot v \left(\frac{\omega^2}{v_i^2} - k^2 \right)^{\frac{1}{2}}.$$

where $C_{12} = (-\bar{C}_1)_y$, \bar{C}_1 is arbitrary complex vector chosen such that $\nabla \cdot \bar{\psi} = 0$, k is an arbitrary complex number such that $k_{\text{Re}} \geq 0$ to ensure propagation in the positive x-direction. The subscripts

Re and Im denote the real and imaginary parts of the corresponding complex quantities. Following Borchardt (1982), the displacement potentials given by (13) to (17) satisfy the boundary conditions for all values of x provided that

$$k = k_{\text{Re}} = \frac{\omega \sin \theta_0}{\alpha \text{ or } \beta} = \frac{\omega \sin \theta_1}{\alpha} = \frac{\omega \sin \theta_2}{\beta} = |\bar{P}_{12}| \sin \theta_1^* = |\bar{P}_{22}| \sin \theta_2^* = |\bar{P}_{32}| \sin \theta_3^*, \quad (18)$$

$$k_{\text{Im}} = |\bar{A}_{j2}| \sin(\theta_j^* - \gamma_j), \quad (j = 1, 2, 3) \quad (19)$$

Which is the extension of Snell's law. We also obtain the following non-homogeneous system of five equations

$$\sum_{j=1}^5 a_{ij} Z_j = b_i, \quad (i = 1, 2, \dots, 5), \quad (20)$$

where

$$a_{11} = -\lambda k^2 - (\lambda + 2\mu)(d\alpha)^2, \quad a_{12} = 2\mu k d\beta,$$

$$a_{13} = (H^* - \alpha^* M^*) (dv_1)^2 + k^2 \left\{ \lambda^* + \alpha^{*2} M^* - \alpha^* M^* \cdot (1 - \mu_1) - \mu_1 M^* \right\} + \mu_1 (\alpha^* M^* - M^*) \cdot (dv_1)^2,$$

$$a_{14} = (H^* - \alpha^* M^*) (dv_2)^2 + k^2 \left[\lambda^* + \alpha^{*2} M^* - \alpha^* M^* (1 - \mu_2) - \mu_2 M^* \right]$$

$$\begin{aligned}
 & +\mu_2(\alpha^* M^* - M^*)(dv_2)^2, \\
 a_{15} & = kdv_3(H^* - \lambda^* - \alpha^* M^*), \\
 a_{21} & = -2\mu k d\alpha, \quad a_{22} = -\mu[(d\beta)^2 - k^2], \quad a_{23} = -2\mu^* k dv_1, \\
 a_{24} & = -2\mu^* k dv_2; \quad a_{25} = \mu^*[(dv_3)^2 - k^2] \\
 a_{31} & = d\alpha, \quad a_{32} = -k, \quad a_{33} = dv_1, \quad a_{34} = dv_2, \quad a_{35} = k, \\
 a_{41} & = 0, \quad a_{42} = 0, \quad a_{43} = -(1 - \mu_1)dv_1, \quad a_{44} = -(1 - \mu_2)dv_2, \\
 a_{45} & = -(1 - \alpha_0)k, \\
 a_{51} & = ik\tau_0; \quad a_{52} = id\beta\tau_0; \quad a_{53} = 2kdv_1\mu^* - ik\tau_0; \\
 a_{54} & = 2kdv_2\mu^* - ik\tau_0; \quad a_{55} = i\tau_0dv_3 - \mu^*\{(dv_3)^2 - k^2\} \\
 a_{51} & = k^2 \frac{\psi}{1-\psi} \cdot \frac{1}{\sin\theta_0}, \quad a_{52} = kd\beta \frac{\psi}{1-\psi} \cdot \frac{1}{\sin\theta_0} \\
 a_{53} & = 2kdv_1 - k^2 \frac{\psi}{1-\psi} \cdot \frac{1}{\sin\theta_0}, \quad a_{54} = 2kdv_2 - k^2 \frac{\psi}{1-\psi} \cdot \frac{1}{\sin\theta_0}, \\
 a_{55} & = kdv_3 \frac{\psi}{1-\psi} \cdot \frac{1}{\sin\theta_0} - \{(dv_3)^2 - k^2\},
 \end{aligned}$$

(a) For incident P wave,

$$b_1 = -a_{11}, \quad b_2 = a_{21}, \quad b_3 = a_{31}, \quad a_{41} = b_4, \quad b_5 = -a_{51},$$

and

$$Z_1 = \frac{A_1}{A_0}, \quad Z_2 = \frac{A_2}{A_0}, \quad Z_3 = \frac{B_{11}}{A_0}, \quad Z_4 = \frac{B_{21}}{A_0}, \quad Z_5 = \frac{C_{12}}{A_0},$$

Are amplitude ratios of reflected P , reflected SV , refracted P_{12} , refracted P_{22} and refracted P_{32} waves, respectively.

a) For Incident SV wave,

$$b_1 = a_{12}, \quad S_v : b_2 = -a_{22}, \quad b_3 = -a_{32}, \quad a_{42} = b_4, \quad b_5 = a_{52};$$

and

$$Z_1 = \frac{A_1}{A_0^*}, \quad Z_2 = \frac{A_2}{A_0^*}, \quad Z_3 = \frac{B_{11}}{A_0^*}, \quad Z_4 = \frac{B_{21}}{A_0^*}, \quad Z_5 = \frac{C_{12}}{A_0^*},$$

Are amplitude ratios of reflected P , reflected SV , refracted P_{12} , refracted P_{22} and refracted P_{32} waves, respectively. For $\psi = 1$, the above system of equations (20) reduces for welded interface.

V. NUMERICAL RESULTS AND DISCUSSION

For numerical computations of reflection and transmission coefficients, we resolve the operators λ^* , μ^* , γ^* and δ^* into their real and imaginary parts, for a general linear viscoelastic solid. Following Biot (1962), the operators γ^* and δ^* are approximated by elastic coefficients, i.e. $\gamma^* = \gamma$, $\delta^* = \delta$. Following Silva (1976), we write $\mu^* = \mu_R (1 + iQ_s^{-1})$, and $\lambda^* + 2\mu^* = (\lambda_R + 2\mu_R)(1 + iQ_e^{-1})$, where Q_e^{-1} and Q_s^{-1} are compressional specific attenuation and shear specific attenuation, respectively. Subscript R denotes the real parts of the corresponding quantities. Following Biot (1956b), Poiseuille flow breaks down if frequency $f (= \omega / 2\pi)$ exceeds a certain value f_t given by $f_t = \frac{\pi\eta}{\rho_f 4d^2}$, where d is the diameter of the pores. If the pores behave like circular tubes, then $f_t f_c = 0.154$, where $f_c = \eta\beta / \chi 2\pi\rho_f$ is the characteristic frequency. Therefore, we are restricted to the frequency range $0 < \frac{f}{f_c} < \frac{f_t}{f_c} = 0.154$. Following Murphy III (1982), we consider water-

saturated Massilon-sandstone with the following parameters: Porosity = 23 per cent, Grain density = 2.66 gm/cm³. Pore diameter = 3×10^{-3} cm. Following Biot (1956b), in case of water in the pores at 15°C, we find $f_t = \frac{10000}{9}$ Hertz for $d = 3 \times 10^{-3}$ cm. Specific attenuation Q_e^{-1} and Q_s^{-1} in partially Massilon-sandstone are strongly frequency dependent. However, dependence is weak only in very dry rocks or in wetted rocks (100 per cent saturation). Following Murphy III (1982) for 100 per cent water-saturated Massilon-sandstone, we choose $Q_e^{-1} = 0.04$ and $Q_s^{-1} = 0.047$ at $f \sim 560$ Hz. Hence we find $\frac{f}{f_c} = 0.0776$ approx, and $\frac{\eta}{\omega\chi} = \frac{\rho_t}{\beta \frac{f}{f_c}}$, where ρ_f is the density of interstitial water and is assumed to be 1 gm/cm³. Following Zwikker and Kosten (1949), Biot's parameter m may be expressed as $m = \frac{c\rho_f}{\beta}$; $c \geq 1$, where, for uniform circular pores with axes parallel to the pressure gradient, c would be equal to 1.

Following Fatt (1959) and Yew and Jogi (1976) relevant elastic parameters for water-saturated sandstone are chosen to be

$$\delta = 0.73787 \times 10^{-11} (\text{dynes} / \text{cm}^2)^{-1}, \gamma = 0.889 \times 10^{-11} (\text{dynes} / \text{cm}^2)^{-1}$$

$$\mu_R = 0.922 \times 10^{11} \text{ dynes} / \text{cm}^2, \lambda_R = 0.3032 \times 10^{11} \text{ dynes} / \text{cm}^2.$$

Following Bullen (1963), the material constants of granite as elastic half-space are considered as

$$\lambda = 2.238 \times 10^{11} \text{ dynes} / \text{cm}^2, \mu = 2.992 \times 10^{11} \text{ dynes} / \text{cm}^2, \rho = 2.66 \text{ gm} / \text{cm}^3.$$

Using all the above numerical values and equations (18) and (19), the reflection and transmission coefficients Z_1, Z_2, Z_3, Z_4 and Z_5 , given by (20), are computed for incident P and SV waves. The angle of incidence θ_0 , is considered to be varying from normal incidence ($\theta_0 = 0^\circ$) to grazing incidence ($\theta_0 = 90^\circ$). We restrict the numerical computations for homogeneous case only.

a) Loosely Boundary Effect

i. Incident P wave

The amplitude ratios of reflected P and SV waves for $\psi = 0.25, 0.5, 0.75$ and 1.0 are plotted against the angle of incidence ($0^\circ < \theta_0 < 90^\circ$) of P wave. These variations are shown in Figures 2 and 3 by black, blue, red and green curves, respectively. In each

case, the amplitude ratios of reflected P and SV waves are same at normal and grazing incidence. The comparison of the different curves shows the effect of loose boundary on amplitude ratios of reflected P and SV waves. This effect is observed maximum in the range $45^\circ < \theta_0 < 90^\circ$. The amplitude ratios of refracted P_{12}, P_{22} and P_{32} waves for $\psi = 0.25, 0.5, 0.75$ and 1.0 are plotted against the angle of incidence of P wave. These variations are shown in Figures 4 to 6 by black, blue, red and green curves, respectively. These amplitude ratios are also affected due to loosely boundary at angles other than grazing and normal incidence.

ii. Incident SV wave

The amplitude ratios of reflected P, SV waves and refracted P_{12}, P_{22} and P_{32} waves for $\psi = 0.25, 0.5, 0.75$ and 1.0 are plotted against the angle of incidence ($0 < \theta_0 < 50$) of SV wave also. These variations are

shown in Figures 7 to 11 by black, blue, red and green curves, respectively. The comparison of the different curves shows the effect of loosely boundary on amplitude ratios of reflected and refracted waves.

b) *Viscoelastic effect*

To observe the viscoelastic effect on reflected and transmitted coefficients, we consider the incidence of P wave and $\nu = 0.25$. On comparing the solid and dotted curves in Figures 12 to 16, it can be seen that the coefficients of reflected and transmitted waves change due to viscoelastic effect.

VI. CONCLUDING REMARKS

Relations between reflection and transmission coefficients are obtained for incident of P and SV at a loosely bonded interface between an elastic solid half-space and a viscoelastic porous solid half-space. Numerical values of these coefficients are computed for a particular model of the interface. It is observed that these coefficients are affected significantly due to the presence of loosely boundary. These coefficients are also affected due to the presence of viscoelasticity in upper half-space.

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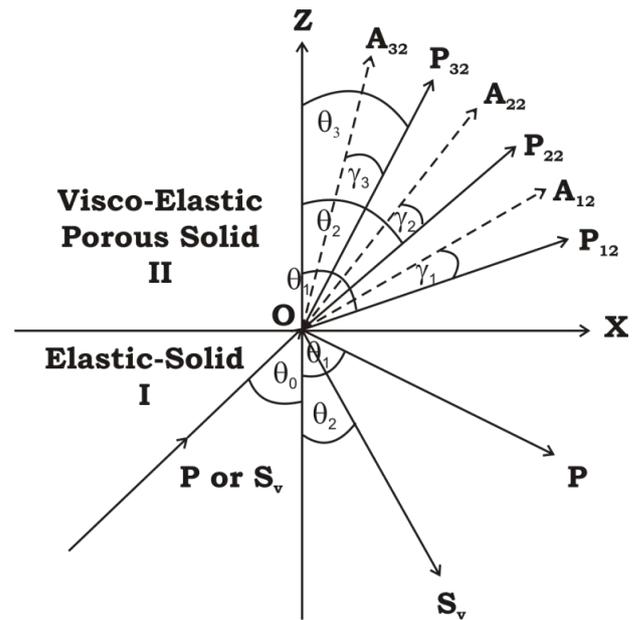


Figure 1 : Geometry of the problem

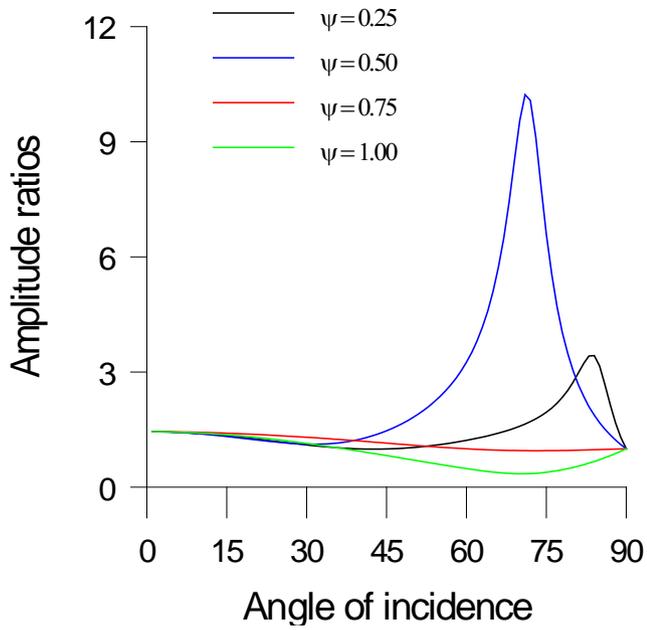


Figure 2 : Variations of the amplitude ratios of reflected P wave against the angle of incidence of P wave for different values of loosely bonded parameter.

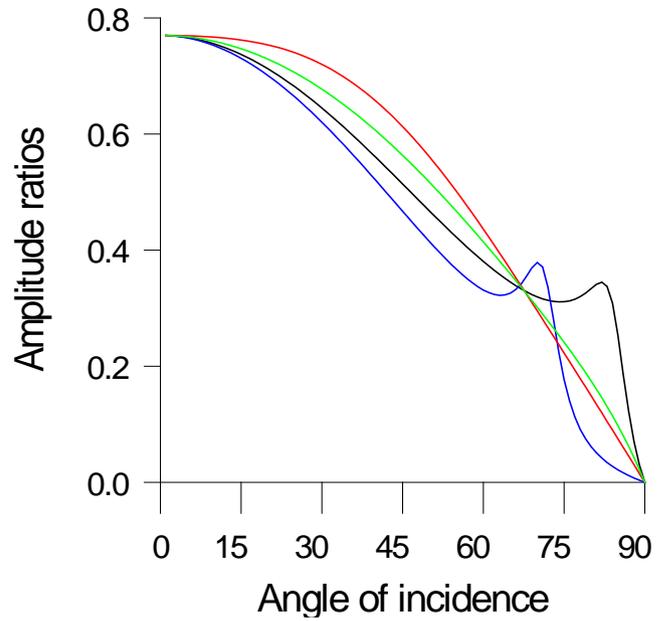


Figure 4 : Variations of the amplitude ratios of reflected P₁₂ wave against the angle of incidence of P wave for different values of loosely bonded parameter

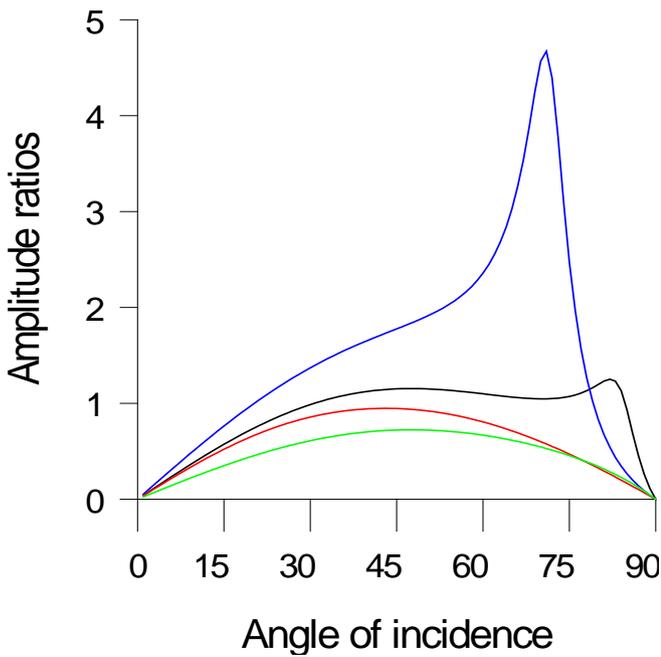


Figure 3 : Variations of the amplitude ratios of reflected SV wave against the angle of incidence of P wave for different values of loosely bonded parameter

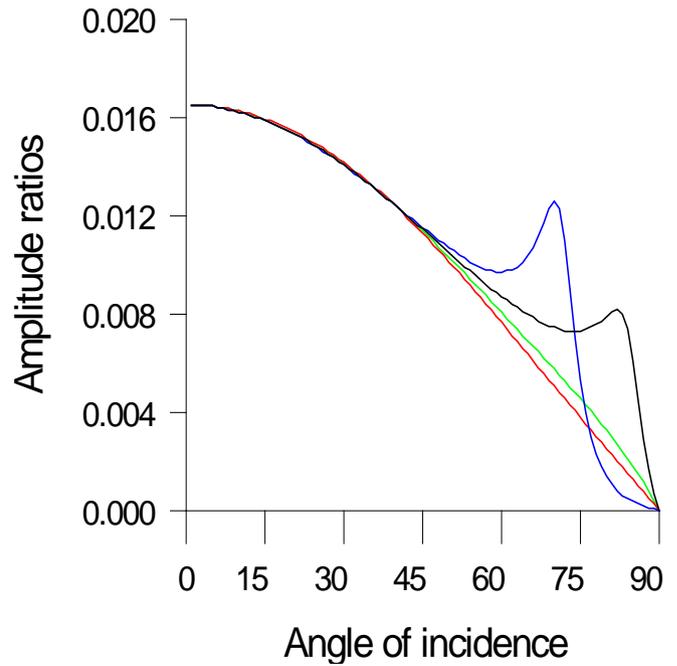


Figure 5 : Variations of the amplitude ratios of reflected P₂₂ wave against the angle of incidence of P wave for different values of loosely bonded parameter

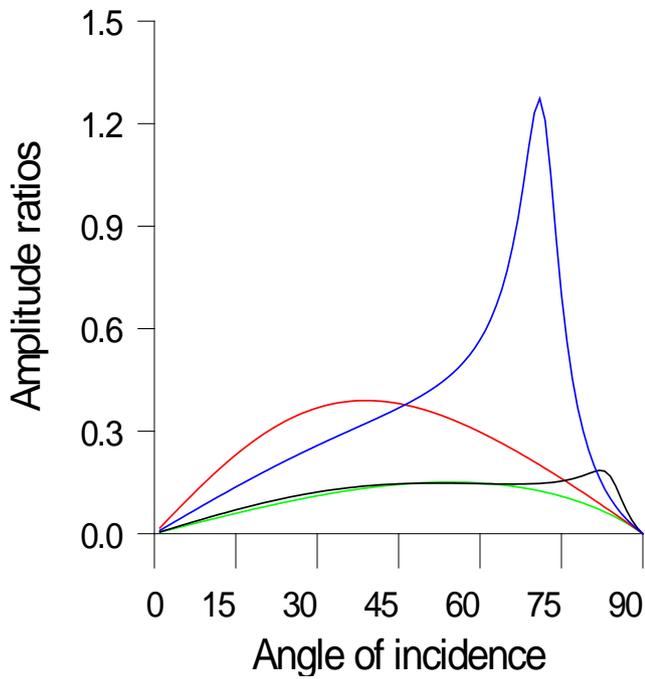


Figure 6 : Variations of the amplitude ratios of reflected P_{32} wave against the angle of incidence of P wave for different values of loosely bonded parameter

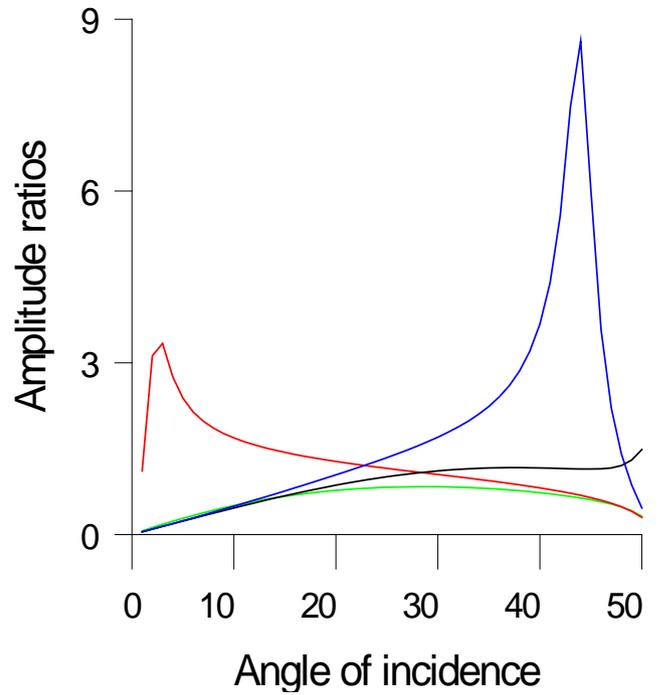


Figure 8. Variations of the amplitude ratios of reflected SV wave against the angle of incidence of SV wave for different values of loosely bonded parameter.

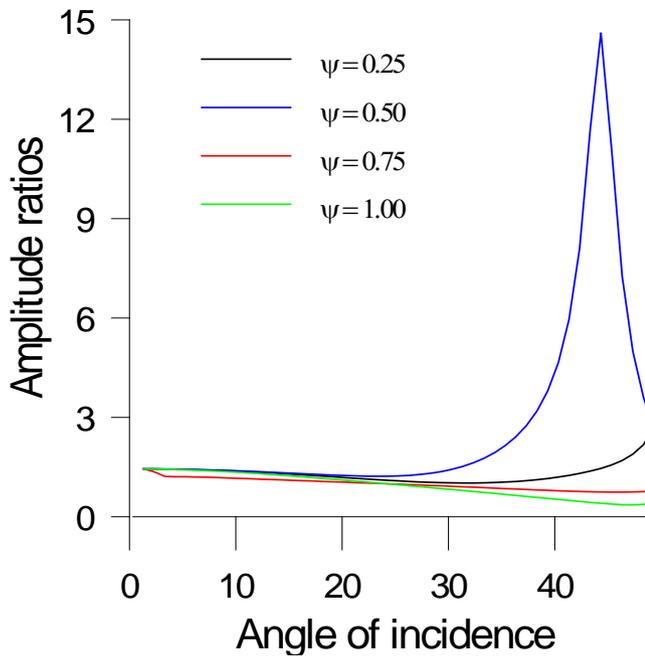


Figure 7 : Variations of the amplitude ratios of reflected P wave against the angle of incidence of SV wave for different values of loosely bonded parameter

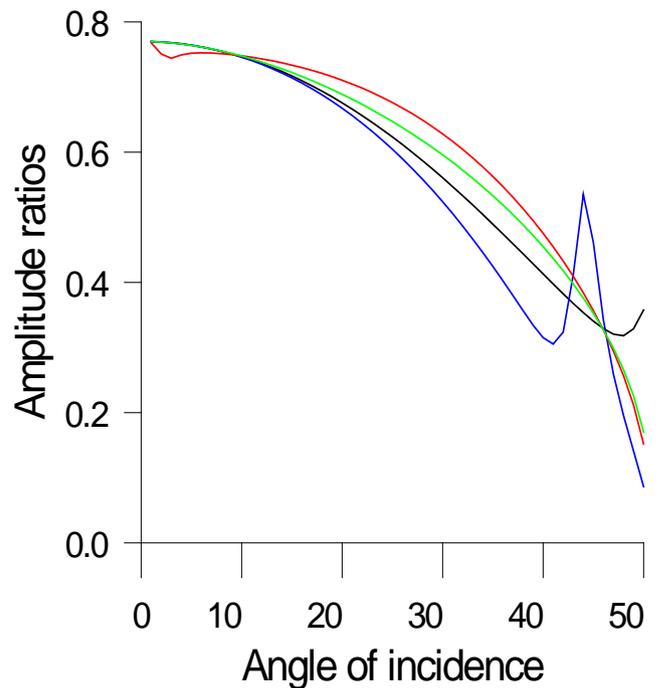


Figure 9 : Variations of the amplitude ratios of reflected P_{12} wave against the angle of incidence of SV wave for different values of loosely bonded parameter

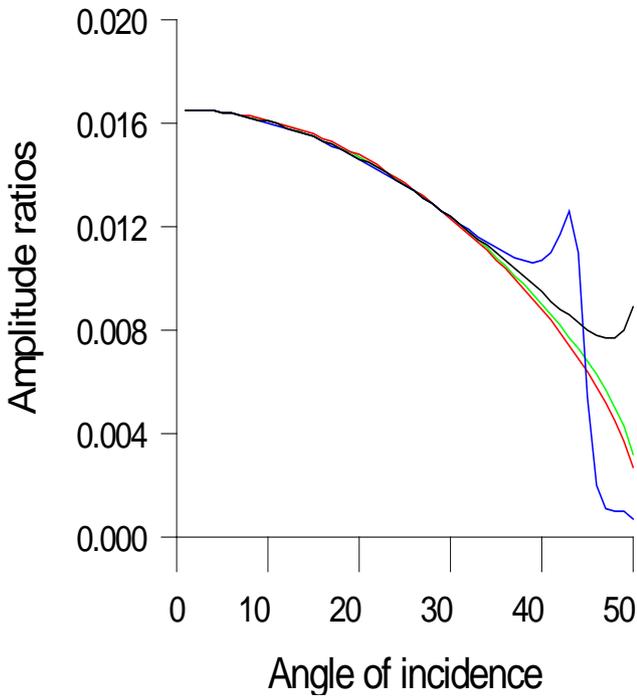


Figure 10 : Variations of the amplitude ratios of reflected P_{22} wave against the angle of incidence of SV wave for different values of loosely bonded parameter

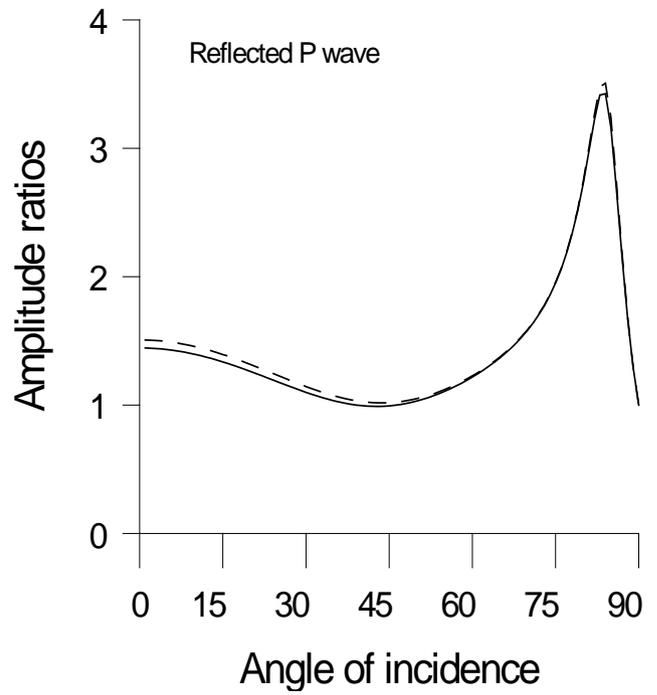


Figure 12 : Viscoelastic effect on the amplitude ratios of reflected P wave against the angle of incidence of P wave for $\psi = 0.25$

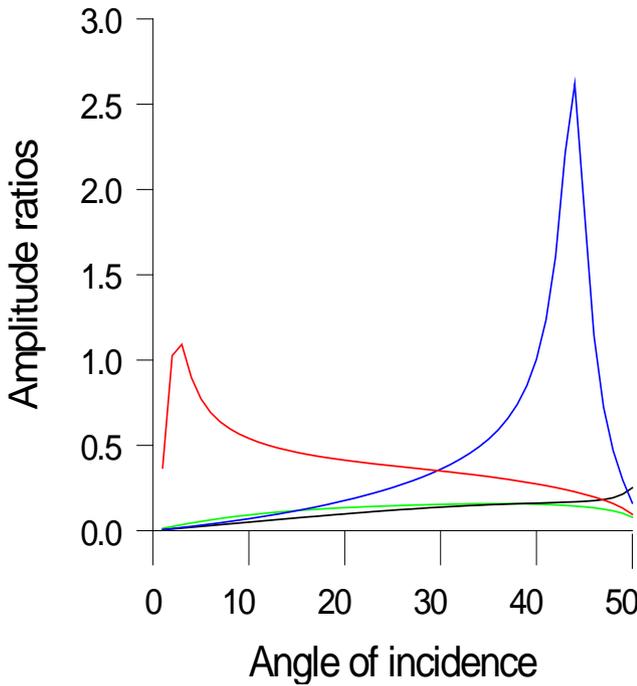


Figure 11 : Variations of the amplitude ratios of reflected P_{32} wave against the angle of incidence of SV wave for different values of loosely bonded parameter

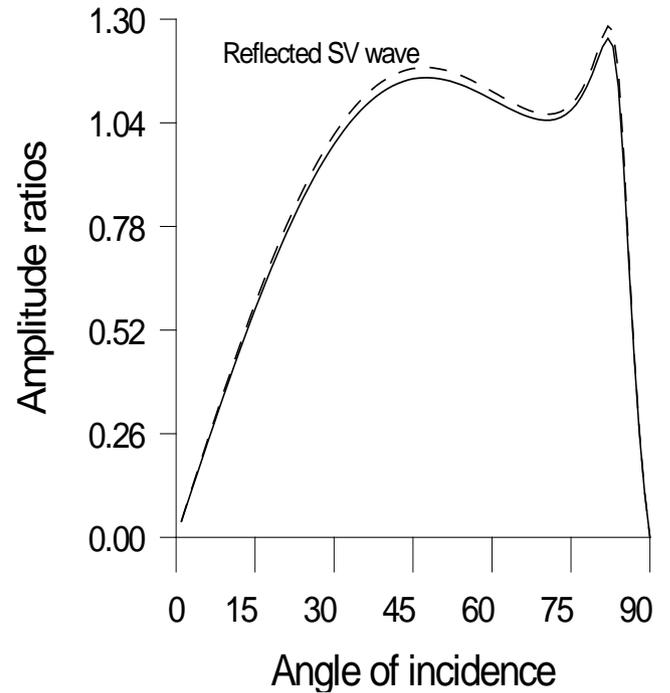


Figure 13 : Viscoelastic effect on the amplitude ratios of reflected SV wave against the angle of incidence of P wave for $\psi = 0.25$

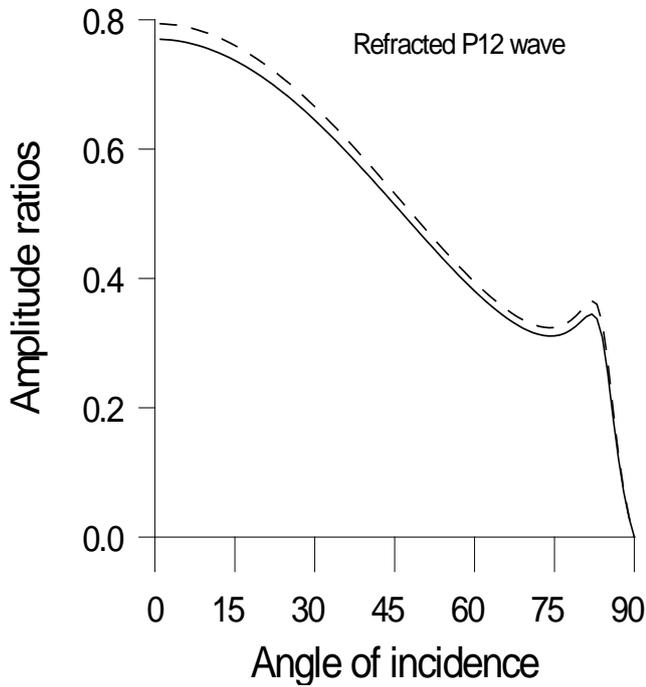


Figure 14 : Viscoelastic effect on the amplitude ratios of transmitted P_{12} wave against the angle of incidence of P wave for $\psi = 0.25$

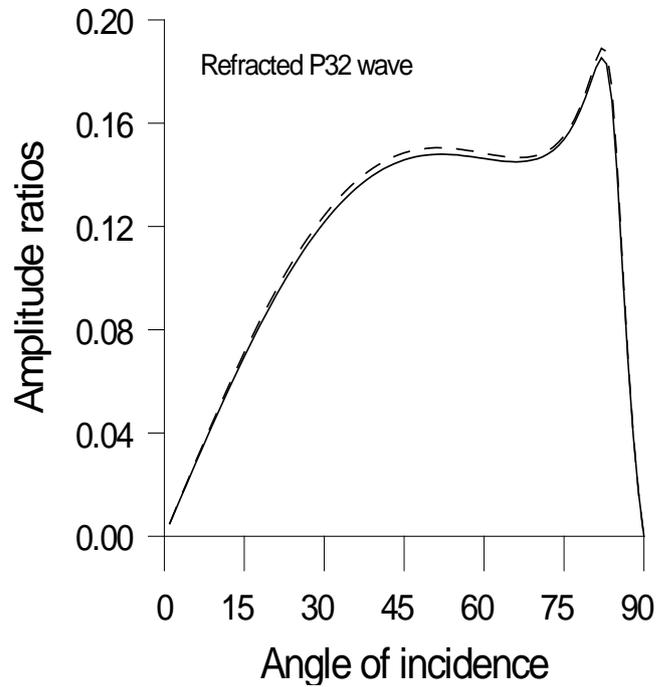


Figure 16 : Viscoelastic effect on the amplitude ratios of transmitted P_{32} wave against the angle of incidence of P wave for $\psi = 0.25$

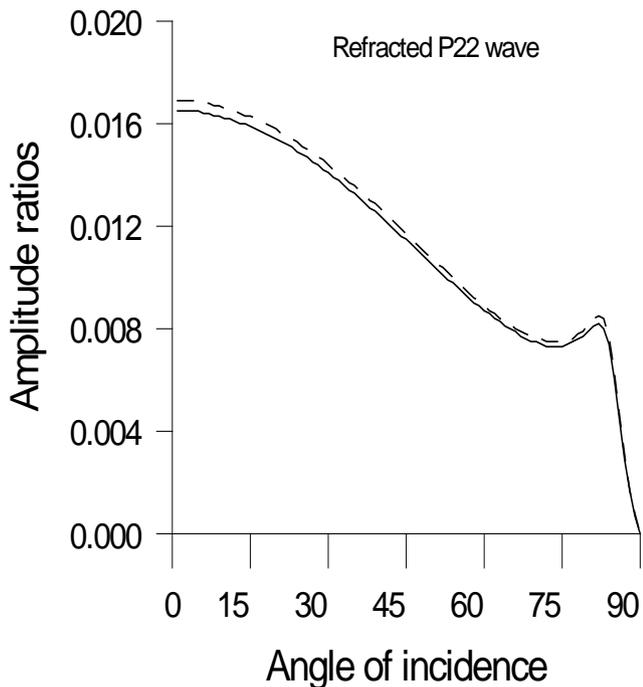


Figure 15 : Viscoelastic effect on the amplitude ratios of transmitted P_{22} wave against the angle of incidence of P wave for $\psi = 0.25$

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