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Comparison of Capability Analysis of Cumulative Cardiac Thoracic Ratio (CTR) Outputs

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I. Introduction

Advance knowledge has made the study of process capability analysis not limited to the industry or manufacturing process only but is gaining overwhelming application in other fields of human endeavour especially in medicine for the evaluation of health care performance such as surgical site control, infection rate, response of patient to change in treatment in the hospital, outbreak of epidemic and performance of a forecasting system related to medical studies such as heart false positive radiological examination. This study looks at the process monitoring of CTR output measurements and check its state of stability for abnormality detection.

In medicine, chest radiography is commonly called chest X-ray (CXR). It is a projection of radiography of the chest use to diagnose conditions affecting the chest, its contents and nearly structure. Ribeiro, Jose, Renato, Roberto, Francisco, Domingo, and Beatriz (2012) observed that chest radiography is among the most common films taken to diagnose many conditions. Like all methods of radiography, chest radiography employs ionizing radiation in the form of x-rays to generate images of the chest (Ribeiro, et al. (2012).

This research is motivated by the real life application of process capability analysis in the output of Cardio Thoracic Ratio of chest X-ray measurements in the examination of radiological process to establish capability analysis of the CTR experimental values. The aim of this study is to determine the capability analysis of Radiological CTR experimental values and the simulated values. The Specific objectives of the research are:

- To do capability analysis for experimental (Raw) and Simulated Cardiac Thoracic Ratio (CTR) values.
- To compare the capability analysis of the experimental Cardiac Thoracic Ratio (CTR) data (Raw values) and the simulated Cardiac Thoracic Ratio (CTR) data.
- To examine the significant difference in the variance of Cardiac Thoracic Ratio (CTR) data of raw and simulated CTR values.

II. Literature Review

The most commonly and widely used indices are C_p (Juan 1974), C_{pk} (Kane 1986), C_{pm} (Hsiang and Taguchi 1985) and C_{pmk} (Choiward and Owen 1970; Pearn and Kotz and Chen 1994-95) and their generalization for non-normal process suggested (Pearn and Kotz, 1995; Pearn and Chen 1995). Mukherjee (1995) studied conceptual approaches to process capability analysis. A number of new approaches to process capability analysis have been attempted and experimented (Carr 1991; Flaig 1996). Another index is given by Boyles (1994), when researcher or quality control officer is confronted with processes described by a characteristic whose values are discrete. Therefore, in such cases none of these indices can be used. The indices suggested so far whose assessment is meaningful regardless of whether the studied process in discrete or continuous are those suggested by Yeh and Bhaltachiya (1998). Borges and Ho (2001), Perakis and Xekalaki (2002; 2005) and James (1998) devised control

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charts for clinical process improvement and sample size determination for discrete and continuous processes.

When the process parameters μ and σ are known, a kind of contour plots of the index C_{PK}, called process performance chart, was introduced to understand capability of a process. To also compare the indices c_p, c_{pk} and c_{pmk} (Boyles 1991) used contour plots called (μ, σ) - plots of these indices as functions of the process parameter (μ, σ). Deleryd and Vannman (1999) and Vannman (2001) used contour plots, called (δ, γ) - plots, as functions of the process parameters to illustrate the restrictions that the different indices in the $c_p(u,v)$ – family impose on the process parameter (μ, σ). When the process parameters μ and σ are unknown and need to be estimated, Deleryd and Vannman (1990) and Vannman (2001) developed what they called the (δ, γ) - plot and the confidence rectangle plots. These plots are tools to draw inference about the process capability based on a random studied quality characteristic.

In this study, evaluation of cumulative capability characteristics of the experimental CTR values (Raw) and Simulated CTR values using uniform distribution are investigated.

III. Distribution Process Index Application

In real life application, calculation of proposed capability index boils down to computation of the process yield. To evaluate the process yield, it is necessary to apply a curve fitting method to approximate the quality characteristic distribution $f(x)$. Polansky (1999) used non-parametric approach particularly Kernel density estimation to estimate process yield for both univariate as well as multivariate quality characteristics. Ciarlini, Gigli and Regoliosi (1999) used bootstrap methodology to estimate failed probabilities even in regions not supported by data with accuracy. Independent of the sample variances is useful when data are not nearly normal. The Pearson distribution was implemented (Clement 1989), the Johnson distribution was suggested (Chou and Polansky 1996; Chou, Polansky and Mason 1998; Polansky et al., 1998). Burr distribution was used to describe non-normal process data (Castaghola 1996).

In practice, one may often be faced with processes whose distributions are far from being normal. In this capability study the index and the assumption that the underlying distribution of the examined process is a non-normal form and in particular, exponential. Gunter (1989) observed the experimental distribution arises frequently in industrial processes and were explained in the article (Yeh and Bhattachayya 1998). The normal and exponential process index is achievable for continuous process however; they are useless when the process is discrete. Poison process index C_{pk} is used in the assessment of discrete process. The properties of are examined in the case where the studied process is described by a poison distribution characteristic with parameter m>0. The uniform process index is achievable for continuous process however; it is useful when the process was discrete. Uniform process index C_{pk} is used in the assessment of discrete process. The properties of C_{pk} are examined in the case where the studied process is described by a uniform distribution characteristic with some parameter a and b (Maiti *etal.*, 2009).

In this study chart such as histogram with normal distribution is used to detect the trend behaviour of the CTR distribution outlier for abnormal CTR values. Uniformly simulated data will be compared with the raw CTR values based on capability analysis and variance. Uniform distribution process is simulated to compare with the raw CTR value of chest radiological examination in this study.

IV. Simulation Technique

Simulation provides a method for checking your understanding of the world around you and helps us to produce better results faster.

a) A Study Simulation

In the study of Cardiac Thoracic Ration of Chest X-ray films examination, the raw values of cardiac and thoracic measure shall be computed to obtain the CTR value of patients that undergo the Chest X-ray examination as:

$$CTR = \frac{C_V}{T_V} \quad (1)$$

where C_V is the cardiac value and the T_V is the thoracic value of the measurements. If the CTR=0.5, the reading is said to be normal with boundary allowances of 0.45 and 0.55 for error of readings accommodation. Hence, the tolerance values are USL=0.55 and LSL=0.45 with the target value

$$T = \left(\frac{USL + LSL}{2}\right) = 0.5. \quad (2)$$

The study employs simulation technique using uniform distribution process between $F(b) = 0.71$ and $F(a) = 0.43$ with 5 numbers of subgroups for 150 random numbers making a total of 750 simulated patients' CTR values for the study.

V. Design and Implementation of Simulation

The simulation use in this study follows a uniform distribution process which ranges from 0.43 to 0.71 with 5 number of variable as subgroup measurements for 150 sample random number all together making 750 observations. Excel application package is the implementation medium used for the random number generation.

VI. Variance ctr Raw and Simulated Processes Comparison

Bartlet 'b'-statistic is assumed as test-statistic that is distributed approximately as χ^2 – distribution when samples are independently drawn from normal population (Singha, 2002). We test that $H_0 : \sigma_r^{\,2} = \sigma_s^{\,2}$ and $H_0 : \sigma_r^{\,2} \neq \sigma_s^{\,2}$ to determine equality of variances (Gomez and Kwanchai, 1984) of both raw and simulated CTR values of Chest X-ray measurement. Comparison of the variances of the raw CTR and Simulated CTR value is carried out in this study to investigate the process equality of variances. In this study, the variance of the CTR raw and simulated values are computed and tested for homogeneity based on the Bartlet Test 'b' statistic. The algorithm for the procedure is described by the following algorithm steps (A4).

VII. Research Method

The source of data for the analysis is primary through raw computation and computer simulation using uniform distribution. The raw data are generated through the measurement values of the cardiac and thoracic of films output of Chest X-ray of patients from the radiological machine process. The ratios of the measurements are computed to obtain various CTR values over time. Inspection Coding Sheet (ICS) is used to randomly generate the samples for the study. Limits are set equal to 3sigma as $x \pm 3\hat{\sigma}$ for both upper and lower limit (USL and LSL) and tolerance limit was established by $T \pm 0.05$ for the raw and simulated CTR values. $T \quad 0.5$ is based on the specification criteria for non-sensitivity analysis (specificity) while statistical process control is investigated to address process stability. Capability analysis is performed for the two processes. The pattern of the means of the raw and simulated values are detected using exploratory data Analysis (EDA) approach like normal probability plots, empirical CDF functions and Box-plot. In addition, homogeneity of variance of the two processes is investigated based on Bartlet's 'b' statistic. The analysis of data is performed electronically with the aid of statistical software MINITAB version 16.0.

VIII. Data Analysis and Result

This aspect focuses on exploring data analysis behaviour pattern of Raw and Simulated Cardiac Thoracic Ratio (CTR) values. It also discusses control chart graphs, process capability analysis and the process variance comparison using Bartlet 'b' statistic.

a) Exploratory Data Analysis of RCTRv and SCTRv

The result of normality plots, cumulative probability density and box plot descriptive analysis results are summary as follow.

b) Probability Plot of RCTRv and SCTRv

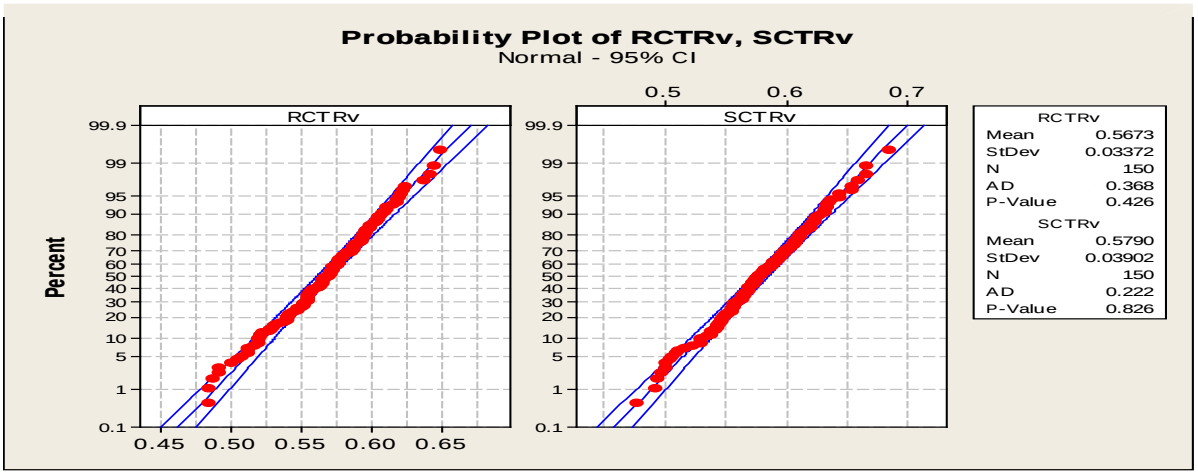

Figure 1 : Normality Plot of RCTRv and SCTRv

The probability plots of raw and simulated CTR values illustrated in the fig. above shows that both the raw and simulated CTR values follow linear pattern demonstrating normality trend with Simulated CTR value perfectly fit in the trend. On the average raw cardio thoracic ratio (RCTR) and simulated CTR values are 0.38 and 0.37 with standard deviation values of 0.033 and 0.039 based on the 150 total samples. The probability

values for RCTRv and SCTRv (0.428 and 0.828) suggest strong evidence of accepting that the raw value and the simulated CTRv are normally distributed as the P-values are greater than 0.05 critical value at 5%.

c) Empirical CDF of RCTRv and SCTRv

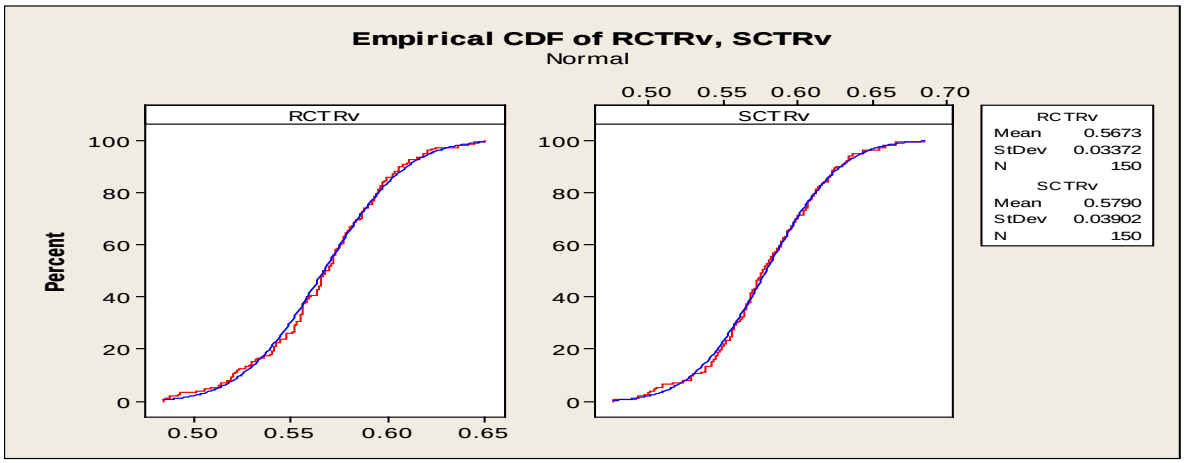

Figure 2 : Cumulative CDF Plot of RCTRv and SCTRv

The probability plots of raw and simulated CTR values illustrated in the fig. above shows that both the raw and simulated CTR values follow linear pattern demonstrating normality trend with Simulated CTR value perfectly fit in the trend. On the average raw cardio thoracic ratio (RCTR) and simulated CTR values are 0.38 and 0.37 with standard deviation values of 0.033 and 0.039 based on the 150 total samples.

d) Boxplot of RCTRv and SCTRv

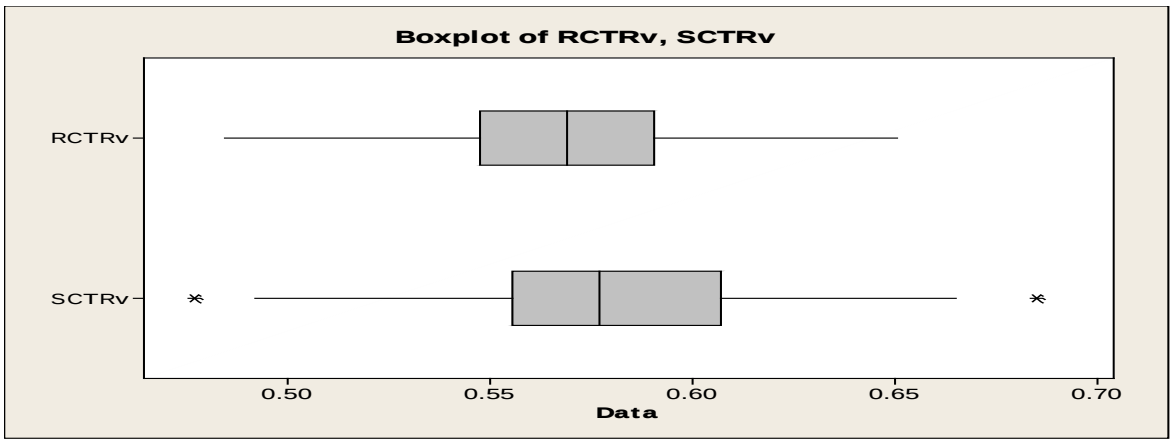

Figure 3 : Boxplot of RCTRv and SCTRv

The Boxplot of RCTRv and SCTRv illustrate non deviation in the RCTRv but deviation exists in the SCTRv because of the existence of the spike (whiskers of dispersion). This confirms that there is likelihood of more deviation from the 0.5 CTR standard in the SCTRv compare to the RCTRv.

e) Process capability Analysis of CTR Measurements of Raw Data N=150

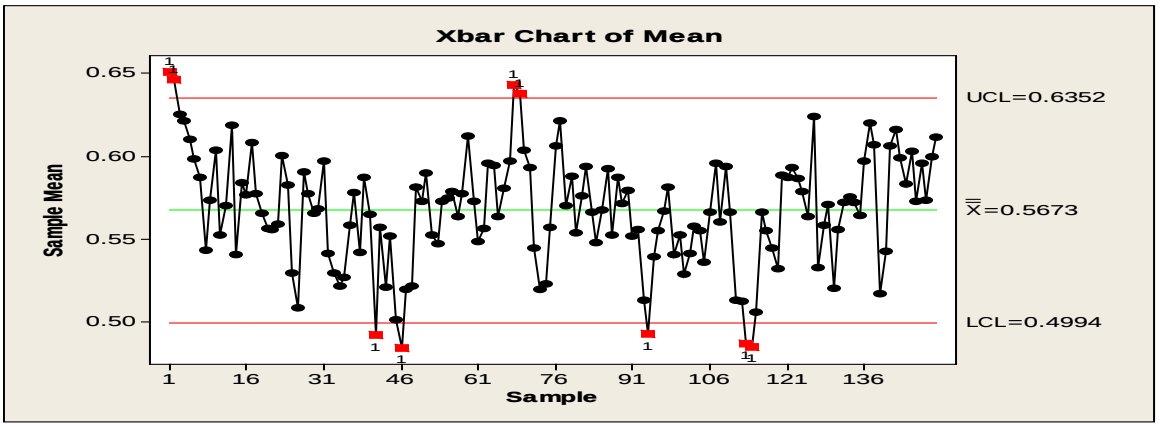

Figure 1a

From the fig1a, the aggregate observation of 150 samples indicates that all points of the raw CTR values are falling within control limit confirming the process statistical stability and under control with predicted trend of sensitivity.

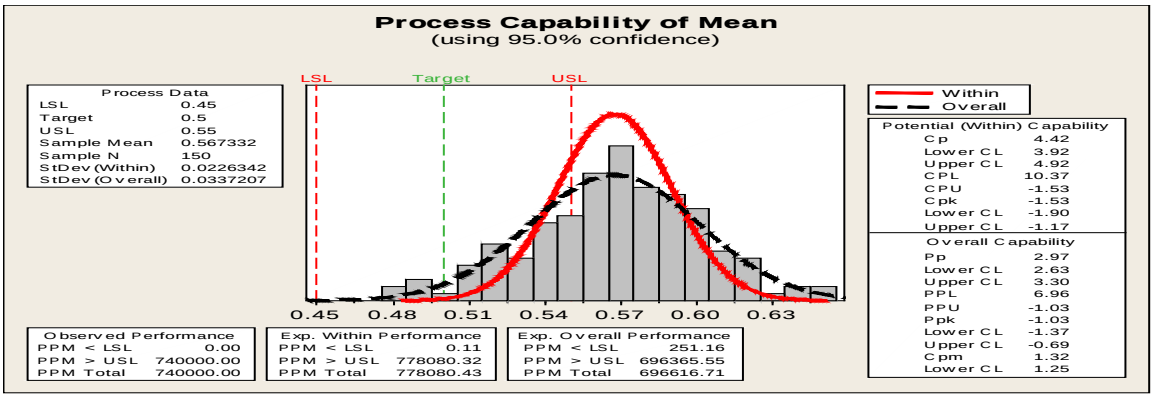

Figure 1b

For sample 150, the mean estimated is 0.5654 where the within and overall standard deviation are 0.0222 and 0.0227, $C_p = 4.42$, $C_{pk} = -1.52$, $C_{pm} = 1.22$ since the $C_{pk} < C_p$, the process is off centred and is toward the lower specification limits. The percentage of the specification band that the process uses up is $P = (1/C_p) * 100 = 22.6\%$. This indicates that the process is using about 22.6% of the specification band.

f) Process capability Analysis Simulated CTR value Uniform DistributionN=150

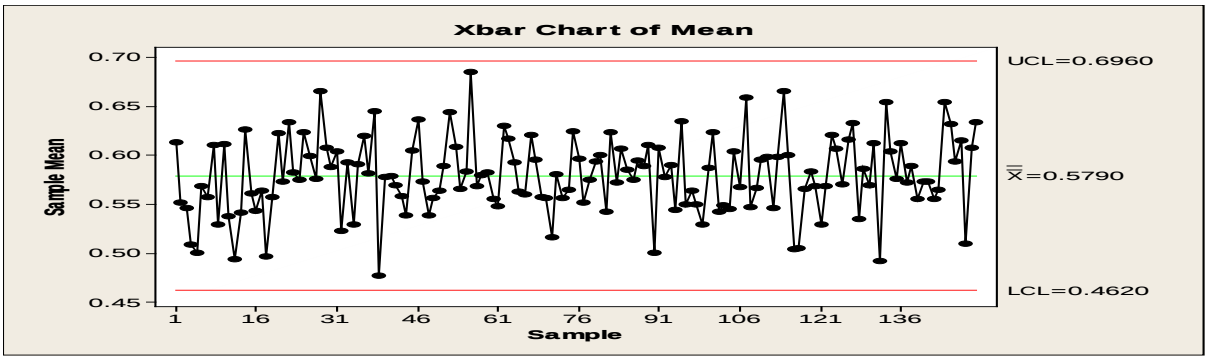

Figure 2a

From the fig2a, cumulative 150 samples all points of the simulated CTR values are falling within control limits implying process stable and follow a predictable trend.

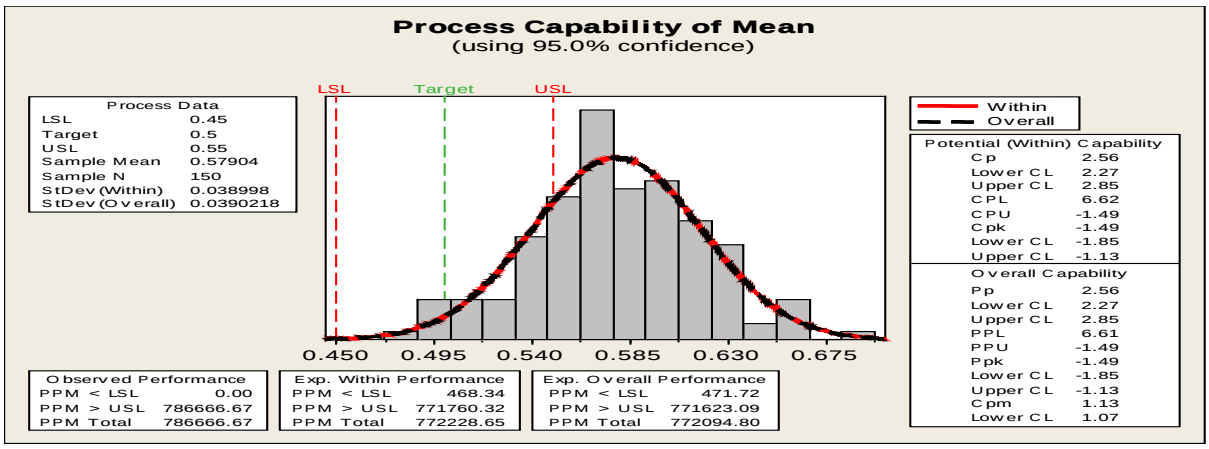

Figure 2b

For cumulative sample 150, the mean estimated is 0.5790 where the within and overall standard deviation are 0.0289 and 0.0290, $C_p = 2.56$, $C_{pk} = -1.49$, $C_{pm} = 1.12$ since the $C_{pk} < C_p$, the process is off centred and is toward the lower specification limits. The percentage of the specification band that the process uses up is $P = (1/C_p)*100 = 39.1\%$. This implies that the process is using about 39.1% of the specification band. Therefore, the values of $C_p = 2.56$ and $P_p = 2.56$ are equal therefore the process has little between subgroup variability. The empirical analysis results and findings of process capability analysis of raw and simulated CTR values are summarized in the table below:

Table 1 : Summary of Findings of cumulative sample process capability analysis
Raw Cardiac Thoracic Ratio Value (RCTRv)

n	25	50	75	100	125	150
μ	0.5654	0.5658	0.5745	0.5681	0.5648	0.5654
σ	0.0323	0.0276	0.0275	0.0226	0.0229	0.0227
C_p	4.42	2.99	4.40	4.27	4.55	4.42
C_{pk}	-2.14	-1.27	-2.16	-1.58	-2.15	-1.52
C_{pm}	1.07	1.21	1.19	1.12	1.26	1.22
P_p	2.09	2.66	2.66	2.97	2.94	2.97
$\left(\frac{1}{C_p}\right)$	0.226	0.334	0.227	0.234	0.2180	0.2260
$P = \left(\frac{1}{C_p}\right)*100$	22.6%	33.4%	22.7%	23.4%	22%	22.6%
$C_p \neq P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$

Source: Results extracted from Minitab 16.0

For cumulative sample 150, the mean estimated is 0.5790 where the within and overall standard deviation are 0.0289 and 0.0290, $C_p = 2.56$, $C_{pk} = -1.49$, $C_{pm} = 1.12$ since the $C_{pk} < C_p$, the process is off centred and is toward the lower specification limits. The percentage of the specification band that the process uses up is $P = (1/C_p)*100 = 39.1\%$. This implies that the process is using about 39.1% of the specification band. Values of C_p and P_p are barely equal hence there is substantial between subgroup variability.

Table 2 : Summary of Findings of cumulative sample process capability analysis Simulated Cardiac Thoracic Ratio Value (SCTRv)

n	25	50	75	100	125	150
μ	0.5651	0.5724	0.5768	0.5771	0.5771	0.5790
σ	0.0422	0.0413	0.0400	0.0280	0.0280	0.0227
C_p	2.21	2.38	2.56	2.62	2.61	2.56
C_{pk}	-2.14	-1.08	-1.28	-1.42	-1.42	-1.49
C_{pm}	1.07	1.19	1.15	1.16	1.16	1.22
P_p	2.09	2.66	2.66	2.97	2.94	2.97
$\left(\frac{1}{C_p}\right)$	0.4516	0.4201	0.3908	0.3821	0.3827	0.3907
$P=\left(\frac{1}{C_p}\right)*100$	45.2%	42%	39.1%	38.2%	38.2%	39.1%
$C_p \neq P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$

Source: Results extracted from Minitab 16.0

For cumulative sample 150, the mean estimate is 0.5790 where the within and overall standard deviation are 0.0289 and 0.0290, $C_p = 2.56$, $C_{pk} = -1.49$, $C_{pm} = 1.12$ since the $C_{pk} < C_p$, the process is off centred and is toward the lower specification limits. The percentage of the specification band that the process uses up is $P = (1/C_p)*100 = 39.1\%$. This implies that the process is using about 39.1% of the specification band. Values of C_p and P_p are barely equal hence there is substantial between subgroup variability.

For the total sample 150, the mean value estimated is 0.5790 where the within and overall standard deviation are 0.0289 and 0.0290, $C_p = 2.56$, $C_{pk} = -1.49$, $C_{pm} = 1.12$ since the $C_{pk} < C_p$,the process is off centred and is toward the lower specification limits. The percentage of the specification band that the process uses up is $P = (1/C_p)*100 = 39.1\%$ .This implies that the process is using about 39.1% of the specification band. Hence, the values of $C_p = 2.56$ and $P_p = 2.56$ and are equal then the process has no subgroup variability. The average estimated value of CTR is 0.57 which is 0.02 higher than the upper specification limit. True sensitivity analysis value of about 59.9% is confirmed fail points among the examined patients while the deviation among the sample measures is 0.023. Both C_p and P_p are near approximate hence there is little between subgroup variability.

g) Bartlet Test 'b' Statistic Computation and Result

The computational result of the Bartlet Test 'b' Statistic value do not exceed the Chi-square value, the variance of the raw and the simulated CTR values have unequal variance.

IX. Conclusion

After aggregating all the raw computed CTR values and simulated CTR values obtained, it is empirically confirmed that the system is operating under 1.0 – 1.3 sigma level for the raw CTR values. Around 28-39% of the raw CTR values obtained are falling outside the specification limits and 30-45% of the specification band is being used. In addition, the $C_{pk} < C_p$ for all the cumulative raw CTR values suggesting that the process is off centred and is towards the lower specification limit. Therefore, the points are falling outside the upper specification limit which clearly indicates that the variability in the raw CTR process is very high.

X. Recommendation

Based on the empirical outputs of capability analysis of radiological result of CTR values (raw and simulated), this study therefore recommends that health awareness campaign on slow death resulting from heart failure as a result of absence of early detection of

abnormal CTR value among patients should be created by the government and health agencies. Patients should be medically advised on the measure to control and maintain stable CTR. Also on how to adopt better management methods which can subsequently prevent possibility of high CTR and further study should be conducted on large repeated experimental scale to ascertain the reliability of this study. Fellow up study of patients should be undertaken by the cardiologist to reduce the possible health risk that could result from the CTR.

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Appendix

A1: Algorithm Statistical Quality Control Process

Step 1 : Obtain the data design in sample subgroups in row (i) and Column (j) respectively Let subg $i = 1, 2, 3, 4, ...n.$ for each sample and subg $j = 1, 2, 3, 4, ...m.$ for each sample subgroups respectively.

Step 2 : Calculated the row total values $\sum_{i=1}^{n} x_i$ and the row average value of the sample subgroups and the mean of the mean of sample subgroup as:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i \text{ and } \bar{\bar{X}} = \frac{1}{M}\sum_{j=1}^{M} \bar{X}_j$$

Step 3 : Calculate the sample range and the sample subgroup range;

$$R_i = MaxValue\,of\ x_i - MinValue\,of\ x_i \text{ and } \bar{R}_j = \frac{1}{M}\sum_{j=1}^{M} R_j$$

Step 4 : Compute the sample variance and standard deviation

$$\hat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{\bar{X}}\right)^2}$$

Step 5 : Evaluate the limits USL, CL and LSL for the sample mean

$$USL = \bar{\bar{X}} + A_2\,\bar{R} \qquad LSL = \bar{\bar{X}} - A_2\,\bar{R} \qquad CL = \bar{\bar{X}}$$

Step 6 : Evaluate the limits USL, LSL and CT for the sample range for = 0.577, , when n=5 from the SQC table readings.

$$USL = D_4\,\bar{R} \qquad LSL = D_3\,\bar{R} \qquad CL = \bar{R}$$

for $A_2 =$ 0.577, $D_3 = 0.00$, $D_4 = 2.115$ when n=5 from the SQC table readings.

Step 7 : Plot the graph of $\bar{\bar{X}}$ chart and $\bar{R}$ chart using the step 6 and 7for the plotting conditions against the sample subgroup.

A2: Algorithm Process Capability Analysis

Step 1 : obtain the values of LSL and USL

Step 2 : Compute C_{pu} and C_{pl} as

$$C_{pu} = \frac{USL - \mu}{3\sigma} \text{ and } C_{pl} = \frac{\mu - LSL}{3\sigma}$$

Step 3 : Calculate the $C_p = \frac{USL - LSL}{6\sigma}$

Step 4 : Compute

$$C_{pk} = Min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\}$$

$$= Min\left(C_{pu}, C_{pl}\right)$$

$$\hat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{\bar{X}}\right)^2}$$

Step 5 : Calculate

$$T = (USL + LSL)/2 \quad \mu = \bar{\bar{X}} = \frac{1}{M}\sum_{j=1}^{M} \bar{X}_j$$

Step 6 : Define $k = \frac{|T - \mu|}{(USL - LSL)/2}$

Step 7 : Evaluate $C_{pk} = C_p(1 - k)$

When $LSL \le USL,\ 0 \le k \le 1,\ k = 0$ i.e the process average equals target value T, then $C_{pk} = C_p$ if $\mu = LSL$ or $\mu = USL$ then $k = 1$ and $C_{pk} = 0$

Step 8 : If $T = \dfrac{(LSL - USL)}{2}$, we compute

$$C_{pm} = \frac{USL - LSL}{6\sigma} = \frac{USL - LSL}{6\sigma\sqrt{1 + \left(\dfrac{\mu - T}{\sigma}\right)^2}}$$

Step 9 : If $T \neq \dfrac{(LSL - USL)}{2}$, compute

$$C^*_{pm} = Min\left\{\frac{USL - T}{3\sigma'}, \frac{T - LSL}{3\sigma'}\right\}$$

$$\hat{C}_{pm} = \frac{USL - LSL}{6\sqrt{s^2 + \left(\bar{X} - T\right)^2}}$$

A3 Simulation Algorithms

Step 1 : Open Excel Window

Step 2 : Click on Data> Select Data Analysis

Step 3 : Click on Random Number Generation

Step 4 : Type 5 in the number of variable column then 150 in the number of random column

Step 5 : Click on the distribution combo box and select uniform in the pull down menu

Step 6 : Type in the between column 0.43 and 0.71 values

Step 7 : Click Ok

Step 8 : The simulated CTR value appears in row and column format End.

A4: Algorithm Bartlet Test 'b' Statistic

Step 1 : obtain the values of Raw and Simulated CTR values

Step 2 : Compute s_1^2 and s_2^2 variances of the CTR

$$s_1^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2 \text{ and } s_2^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$$

Step 3 : Calculate the $S^2 = \dfrac{\sum(n_i - 1)S_i^2}{N - K}$ $i = 1,2\ldots$

Step 4 : Compute

$$Q = (n - k)\log S^2 - \sum(n_i - 1)\log S_i^2$$

Step 5 : Calculate

$$H = 1 + \tfrac{1}{3}(k - 1)\left[\sum\frac{1}{(n_i - 1)} - \frac{1}{(N - K)}\right]$$

Step 6 : Compute b

A4 : Bartlet 'b' Statistic computational results for Raw and Simulated CTR values

Raw CTR	Variance	Simulated CTR	Variance
s_1^2	0.674	s_2^2	1.193
$\log s_1^2$	-0.172	$\log s_2^2$	0.0765
$S^2 = \dfrac{\sum(n_i - 1)S_i^2}{N - K} = \dfrac{(n_1 - 1)S_2^2 + (n_2 - 1)S_2^2}{N - K}$ $= \dfrac{(5-1)(0.674) + (5-1)(1.193)}{1500 - 5} = 0.0515$			
$Q = (n - k)\log S^2 - \sum(n_i - 1)\log S_i^2 = (5-2)\log(0.0515) - [(4)(0.674) + (4)(1.193)]$ $Q = (3)(-1.2882) - [7.4675] =$ -11.3321			
$H = 1 + \tfrac{1}{3}(k-1)\left[\sum\dfrac{1}{(n_i - 1)} - \dfrac{1}{(N - K)}\right] = 1 + \tfrac{1}{3}(2-1)\left[\dfrac{1}{(4)(4)} - \dfrac{1}{(150 - 5)}\right]$ $H = 1 + \tfrac{1}{3}\left[\dfrac{1}{16} - \dfrac{1}{(145)}\right] = 1.33(0.0625 - 0.006896) =$ 0.07395			
$b = 2.3026\dfrac{Q}{H} = 2.3026\left(\dfrac{-11.3321}{0.07395}\right) =$ -352.85			

χ_{α}^{2} with $(k-1) = \chi^{2}_{0.05,(2-1)} = 3.38$
Decision Rule * If b value exceeds χ^{2} value, both processes have equal variance. * If b value does not exceed χ^{2} value, both processes do not have equal variance.

Raw Computed Data										Simulated Cardio-Thoracic Ratio Using Uniform Distribution									
Radiographic Films Readings of Chest X-ray										Radiographic Films Readings of Chest X-ray									
For congestive Heart Failure Cardiomegaly Conditions										For congestive Heart Failure Cardiomegaly Conditions									
Cadio-Thoracic Ratio									Variance	Cadio-Thoracic Ratio									
Sample	Sub 1	Sub 2	Sub 3	Sub 4	Sub 5	Total	Mean	Range		Sample	Sub 1	Sub 2	Sub 3	Sub 4	Sub 5	Total	Mean	Range	Variance
1	0.62	0.47	0.71	0.7	0.76	3.25	0.65	0.29	0.013	1	0.64	0.65	0.59	0.66	0.52	3.07	0.61	0.14	0.003
2	0.65	0.7	0.73	0.59	0.56	3.23	0.65	0.17	0.005	2	0.65	0.57	0.64	0.47	0.43	2.76	0.55	0.21	0.010
3	0.67	0.51	0.74	0.57	0.63	3.13	0.63	0.23	0.008	3	0.55	0.64	0.56	0.54	0.44	2.73	0.55	0.19	0.005
4	0.51	0.63	0.78	0.68	0.52	3.11	0.62	0.27	0.013	4	0.6	0.56	0.43	0.48	0.48	2.55	0.51	0.17	0.005
5	0.72	0.53	0.6	0.69	0.51	3.05	0.61	0.2	0.009	5	0.64	0.42	0.47	0.42	0.54	2.5	0.5	0.22	0.009
6	0.63	0.52	0.73	0.55	0.57	2.99	0.6	0.21	0.007	6	0.45	0.47	0.63	0.68	0.61	2.84	0.57	0.22	0.010
7	0.6	0.56	0.71	0.61	0.46	2.94	0.59	0.25	0.008	7	0.49	0.68	0.45	0.55	0.62	2.79	0.56	0.23	0.009
8	0.53	0.52	0.51	0.59	0.57	2.71	0.54	0.09	0.001	8	0.48	0.67	0.72	0.64	0.54	3.05	0.61	0.24	0.010
9	0.54	0.63	0.68	0.54	0.48	2.87	0.57	0.2	0.006	9	0.45	0.51	0.54	0.44	0.7	2.65	0.53	0.26	0.011
10	0.71	0.49	0.64	0.61	0.56	3.02	0.6	0.21	0.007	10	0.62	0.7	0.52	0.51	0.72	3.06	0.61	0.22	0.010
11	0.61	0.48	0.5	0.52	0.66	2.76	0.55	0.18	0.006	11	0.64	0.62	0.51	0.46	0.47	2.69	0.54	0.18	0.007
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.			.		.			.	.	.		.	.	.	.	.	.	.	.
137	0.59	0.62	0.64	0.72	0.53	3.1	0.62	0.19	0.005	137	0.55	0.54	0.68	0.47	0.62	2.86	0.57	0.21	0.006
138	0.66	0.62	0.61	0.58	0.57	3.03	0.61	0.08	0.001	138	0.53	0.67	0.73	0.48	0.54	2.95	0.59	0.25	0.011
139	0.57	0.5	0.46	0.52	0.53	2.59	0.52	0.11	0.002	139	0.68	0.62	0.51	0.43	0.53	2.78	0.56	0.24	0.010
140	0.6	0.57	0.55	0.5	0.5	2.71	0.54	0.1	0.002	140	0.56	0.58	0.59	0.54	0.6	2.86	0.57	0.06	0.001
141	0.58	0.57	0.74	0.64	0.5	3.03	0.61	0.24	0.008	141	0.47	0.65	0.61	0.63	0.5	2.87	0.57	0.18	0.007
142	0.66	0.54	0.71	0.66	0.51	3.08	0.62	0.2	0.007	142	0.51	0.54	0.61	0.66	0.46	2.77	0.55	0.2	0.006
143	0.6	0.58	0.61	0.68	0.52	3	0.6	0.16	0.003	143	0.67	0.55	0.55	0.46	0.59	2.82	0.56	0.21	0.006
144	0.63	0.48	0.55	0.67	0.58	2.91	0.58	0.19	0.005	144	0.65	0.69	0.67	0.56	0.7	3.27	0.65	0.15	0.003
145	0.73	0.53	0.56	0.67	0.52	3.01	0.6	0.21	0.009	145	0.56	0.7	0.71	0.6	0.59	3.16	0.63	0.15	0.005
146	0.73	0.5	0.56	0.55	0.52	2.86	0.57	0.23	0.008	146	0.73	0.52	0.53	0.49	0.69	2.97	0.59	0.24	0.012
147	0.66	0.52	0.57	0.62	0.62	2.98	0.6	0.15	0.003	147	0.46	0.63	0.71	0.55	0.72	3.07	0.61	0.26	0.012
148	0.57	0.53	0.59	0.56	0.63	2.87	0.57	0.1	0.001	148	0.54	0.56	0.52	0.44	0.49	2.55	0.51	0.12	0.002
149	0.62	0.57	0.6	0.59	0.61	3	0.6	0.05	0.000	149	0.61	0.47	0.66	0.59	0.7	3.04	0.61	0.23	0.008
150	0.58	0.53	0.59	0.75	0.62	3.06	0.61	0.22	0.007	150	0.71	0.56	0.63	0.57	0.69	3.17	0.63	0.16	0.005

Source: Chest X-ray Radiological Readings 2011

Source: Chest X-ray Radiological Simulation Readings 2011