

Relationship between New Types of Transitive and Chaotic Maps

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Abstract

The concepts of topological τ -transitive maps, τ -transitive maps, τ -type chaotic and τ -type chaotic maps were introduced by M. Nokhas Murad Kaki. In this paper, I study the relationship between two different notions of transitive maps, namely topological τ -transitive maps, topological τ -transitive maps and investigate some of their properties in two topological spaces (X, τ) and (X, τ') , τ denotes the τ -topology (resp. τ' denotes the τ' -topology) of a given topological space (X, τ) . The two notions are defined by using the concepts of τ -irresolute map and τ' -irresolute map respectively. Also, we study the relationship between two new types of chaotic maps, namely, τ -type chaotic maps and τ' -chaotic maps, and I will prove that the properties of τ -transitive, τ -type chaotic are preserved under τ -conjugacy and τ' -transitive, τ' -chaotic maps are preserved under τ' -conjugacy. The main results are the following propositions: 1) Every topologically τ -type transitive map is a topologically τ -type transitive map which implies topologically transitive map, but the converse not necessarily true. 2) Every τ -type chaotic map is τ' -chaotic map which implies chaotic map in topological spaces, but the converse not necessarily true.

Index terms— topologically τ -transitive, τ -type chaotic, τ' -type chaotic, τ -dense.

1 Introduction

Recently there has been some interest in the notion of a locally closed subset of a topological space. According to Bourbaki [16] a subset S of a space (X, τ) is called locally closed if it is the intersection of an open set and a closed set. Ganster and Reilly used locally closed sets in [13] and [14] to define the concept of LC-continuity, i.e. a function $f : (X, \tau) \rightarrow (Y, \tau')$ is LC-continuous if the inverse with respect to f of any open set in Y is closed in X . The study of semi open sets and semi continuity in topological spaces was initiated by Levine [6]. Bhattacharya and Lahiri [8] introduced the concept of semi generalized closed sets in topological spaces analogous to generalized closed sets which was introduced by Levine [5]. Throughout this paper, the word "space" will mean topological space. The collections of semi-open, semi-closed sets and τ -sets in (X, τ) will be denoted by $SO(X, \tau)$, $SC(X, \tau)$ and respectively. Njastad [7] has shown that τ is a topology on X with the following properties: and if and only if N where N is nowhere dense in (X, τ) . Hence if and only if every nowhere dense (nwd) set in (X, τ) is closed, therefore every transitive map implies τ -transitive. Also if every τ -open set is locally closed then every transitive map implies τ -transitive; and this structure also occurs if (X, τ) is locally compact and Hausdorff [36, p. 140, Ex. B] and every τ -open set is locally compact, then every τ -open set is locally closed.

In 1943, Fomin [27] introduced the notion of τ -continuous maps. The notions of τ -open sets, τ -closed sets and τ -closure were introduced by Velicko [19] for the purpose of studying the important class of H -closed spaces in terms of arbitrary fiber-bases. Dickman and Porter [20], [21], Joseph [22] and Long and Herrington [31] continued the work of Velicko. We introduce the notions of τ -type transitive maps, τ -minimal maps and show that some of their properties are analogous to those for topologically transitive maps. Also, we give some additional properties of τ -irresolute maps. We denote the interior and the closure of a subset A of X by $\text{Int}(A)$ and $\text{Cl}(A)$, respectively.

Remark 2.15 For any subset A of the space X , A is $\mathcal{F}$ -closed if and only if $A = \mathcal{F}A$. For subsets A and B of a space $(X, \mathcal{F})$, the following hold: 1) $A \cup B$ is $\mathcal{F}$ -closed; 2) If $A \subseteq B$ then A is $\mathcal{F}$ -closed; 3) If A is $\mathcal{F}$ -closed then $\mathcal{F}A$ is $\mathcal{F}$ -closed; 4) If A is $\mathcal{F}$ -closed then $\mathcal{F}A$ is $\mathcal{F}$ -closed.

5 $\mathcal{F}$ -Compactness

Lemma 2.17 The collection of $\mathcal{F}$ -compact subsets of X is closed under finite unions. If $\mathcal{F}$ is a regular operator and X is an $\mathcal{F}$ - T space then it is closed under arbitrary intersection.

Global Journal of Researches in Engineering () Volume XIV Issue V Version I 26 Year 2014 © 2014 Global Journals Inc. (US) A $\mathcal{F}$ -compact subset of X is a subset A of X such that $A = \mathcal{F}A$. For subsets A and B of a space $(X, \mathcal{F})$, the following hold: 1) $A \cup B$ is $\mathcal{F}$ -compact; 2) If $A \subseteq B$ then A is $\mathcal{F}$ -compact; 3) If A is $\mathcal{F}$ -compact then $\mathcal{F}A$ is $\mathcal{F}$ -compact; 4) If A is $\mathcal{F}$ -compact then $\mathcal{F}A$ is $\mathcal{F}$ -compact.

Relationship between New Types of Transitive and Chaotic Maps Theorem 2.12 Theorem 2.21 [4]: For any subset A of a space X ,

Theorem 2.22. [4]: For subsets A, B of a space X , the following statements are true: 1) $\text{int}(A)$ is the largest $\mathcal{F}$ -open contained in A 2) 3) If 4) 5)

Lemma 2.23 [7] For any $\mathcal{F}$ -open set A and any $\mathcal{F}$ -closed set C , we have 1)

6 $\mathcal{F}$ -Transitivity

Theorem 2.26 Let (X, f) and (Y, g) be two topological systems, if f and g are topologically $\mathcal{F}$ -conjugate. Then (1) f is topologically $\mathcal{F}$ -transitive map if and only if g is topologically $\mathcal{F}$ -transitive map; Remark 2. (2) f is $\mathcal{F}$ -type chaotic map if and only if g is $\mathcal{F}$ -type chaotic map; (3) f is $\mathcal{F}$ -type chaotic map if and only if g is $\mathcal{F}$ -type chaotic map.

7 III.

8 Transitive and Minimal Systems

Topological transitivity is a global characteristic of dynamical systems. By a dynamical system (X, f) [15] we mean a topological space X together with a continuous map $f: X \rightarrow X$. The space X is sometimes called the phase space of the system. A set A is called invariant if $f(A) \subseteq A$. A topological system (X, f) is called minimal if X does not contain any nonempty, proper, closed invariant subset. In such a case we also say that the map itself is minimal. Thus, one cannot simplify the study of the dynamics of a minimal system by finding its nontrivial closed subsystems and studying first the dynamics restricted to them.

Given a point x in X denotes its orbit (by an orbit we mean a forward orbit even if f is a homeomorphism) and $\mathcal{F}x$ denotes its $\mathcal{F}$ -limit set, i.e. the set of limit points of the sequence $\{f^n(x)\}_{n \in \mathbb{N}}$.

The following conditions are equivalent: f is $\mathcal{F}$ -minimal (resp. $\mathcal{F}$ -minimal),

$\mathcal{F}$ every orbit is $\mathcal{F}$ -dense (resp. $\mathcal{F}$ -dense) in X ,

$\mathcal{F}$ for every A minimal map is necessarily surjective if X is assumed to be Hausdorff and compact.

Now, we will study the Existence of minimal sets. Given a dynamical system (X, f) , a set A is called a minimal set if it is non-empty, closed and invariant and if no proper subset of A has these three properties. So, A is a minimal set if and only if A is a minimal system. A system is minimal if and only if X is a minimal set in (X, f) . The basic fact discovered by G. D. Birkhoff is that in any compact system there are minimal sets. This follows immediately from the Zorn's lemma.

Since any orbit closure is invariant, we get that any compact orbit closure contains a minimal set. This is how compact minimal sets may appear in non-compact spaces. Two minimal sets in (X, f) are either disjoint or coincide. A minimal set A is strongly invariant, i.e.

Provided it is compact Hausdorff. Let (X, f) be a topological system, and f $\mathcal{F}$ -homeomorphism of X onto itself. For A and B subsets of X , we let

We write for a singleton thus For a point x we write for the orbit of x and for the $\mathcal{F}$ -closure of x . We say that the topological system (X, f) is $\mathcal{F}$ -type point transitive if there is a point x with $\mathcal{F}x = X$. Such a point is called $\mathcal{F}$ -type transitive. We say that the topological system (X, f) is topologically $\mathcal{F}$ -type transitive (or just $\mathcal{F}$ -type transitive) if the set is nonempty for every pair U and V of nonempty $\mathcal{F}$ -open subsets of X .

Theorem 2.8 [37] Let (X, f) be a topological system where X is a non-empty locally $\mathcal{F}$ -compact Hausdorff topological space and f is $\mathcal{F}$ -irresolute map and that X is $\mathcal{F}$ -type separable. Suppose that f is topologically $\mathcal{F}$ -type transitive. Then there is an element x such that the orbit $\mathcal{F}x$ is $\mathcal{F}$ -dense in X .

9 a) Topologically $\mathcal{F}$ -Transitive Maps

In [35], we introduced and defined a new class of transitive maps that are called topologically $\mathcal{F}$ -transitive maps on a topological space $(X, \mathcal{F})$, and we studied some of their properties and proved some

²© 2014 Global Journals Inc. (US) denoted by $O(x)$, is $\text{dense}(\text{resp. } \text{?dense})$ in X for each $x \in X$. A partial answer will be given in this section. Let us begin with a new definition.



Figure 1:

Relationship between New Types of Transitive and Chaotic Maps IV.

.1 Alpha-Minimal Functions

Associated with this new definition we can prove the following new theorem.

? Theorem 3.1.13 [35]: Let (X, τ) be a topological space and be τ -irresolute map. Then the following statements are equivalent: (1) f is topological τ -transitive map ? Theorem 3.1.14 : [34] Let (X, τ) be a topological space and be τ -irresolute map. Then the following statements are equivalent:

in X .

? Definition 4.1 (τ -minimal) Let X be a topological space and f be τ -irresolute map on X with τ -regular operator associated with the topology on X . Then the dynamical system (X, f) is called τ -minimal system (or f is called τ -minimal map on X) if one of the three equivalent conditions [35] hold:

1) The orbit of each point of X is τ -dense in X .

2) for each $x \in X$? 3) Given $x \in X$ and a nonempty τ -open U in X , there exists $n \in \mathbb{N}$ such that ? Theorem 4.2 [35] For (X, τ) the following statements are equivalent:

.2 Topological Systems and Conjugacy

In this section, I introduce and define τ -conjugated topological systems (X, f) and (Y, g) , where X and Y are almost regular topological spaces. First I will define τ -homeomorphism and then I will prove new theorem associated with these new definitions:

? Definition 5.1 A map is said to be τ -homeomorphism if it is bijective and thus invertible and both f and f^{-1} are τ -irresolute

? Definition 5.2 Two topological systems (X, f) and (Y, g) are said to be almost regular systems if X and Y are almost regular topological spaces.

? Definition [38] 5.3 Let (X, f) and (Y, g) be two almost regular systems, then f and g are said to be topologically τ -conjugate if there is a τ -homeomorphism h such that $h \circ f = g \circ h$. will call h a topological τ -conjugacy. Thus, the two almost regular topological systems with their respective function acting on them share the same dynamics VI.

.3 New Types of Chaos of Topological Spaces

We will give a new definition of chaos for τ -irresolute (resp. τ -irresolute) self map of a locally compact Hausdorff topological space X , so called τ -type chaotic (resp. τ -type chaotic). These new definitions imply John Tylar definition which coincides with Devaney's definition for chaos when the topological space happens to be a metric space, but not conversely.

? Definition 4.1 Let (X, f) be a topological system, the dynamics is obtained by iterating the map. Then, f is said to be τ -type chaotic (resp. τ -type chaotic) on X provided that for any nonempty τ -open (resp. τ -open) sets U and V in X , there is a periodic point p such that $p \in U \cap V$.

? Proposition 4.2 Let (X, f) be a topological system.

The map f is τ -type chaotic (resp. τ -type chaotic) on X if and only if f is τ -type transitive (resp. τ -type transitive) and the set of periodic points of the map f is dense (resp. dense) in X .

Let us prove only for τ -type chaotic Proof: If f is τ -type chaotic on X , then for every pair of nonempty τ -open sets U and V , there is a periodic orbit intersects them; in particular, the periodic points are τ -dense in X . Then there is a periodic point p and with $x \in U$ and $y \in V$ and some positive integer n such that $f^n(x) = y$, so that therefore that is, f is τ -type transitive map.

The τ -type transitivity of f on X implies that for any nonempty τ -open subsets $U, V \subset X$, there is n such that for some $x \in U$, $f^n(x) \in V$. Now define

$W = \bigcup_{n \in \mathbb{N}} f^n(U) \cap V$. Then W is τ -open and nonempty with the property that $f(W) \subset W$. But since the periodic points of f are τ -dense in X , there is a $p \in W$ such that $f(p) = p$. Therefore, $p \in U \cap V$, so that f is τ -type chaotic map.

.4 VII.

.5 Conclusion

We have the following results

? Proposition 7.1. Every topologically τ -type transitive map is a topologically τ -type transitive map which implies topologically transitive map, but the converse not necessarily true.. ? proposition 7.2. Every τ -minimal map is τ -minimal map which implies minimal map, but the converse not necessarily true.. ? Theorem 7.3 Let (X, f) and (Y, g) be two topological systems, if f and g are topologically τ -conjugate. Then (1) f is topologically τ -transitive map if and only if g is topologically τ -transitive map;

(2) f is τ -type chaotic map if and only if g is τ -type chaotic map;

(3) f is τ -type chaotic map if and only if g is τ -type chaotic map.

? Proposition 7.4 Let (X, f) be a topological system.

The map f is τ -type chaotic (resp. τ -type chaotic) on X if and only if f is τ -type transitive (resp. τ -type transitive) and the set of periodic points of the map f is dense (resp. dense) in X .

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