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# <sup>1</sup> Vibrations in Steel-Frame Floors due to Human Activities

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#### 6 Abstract

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Architectural new tendencies along with current market demands are taking engineering 7 design towards the use of flooring systems which can span great distances with a minimum 8 number of columns allowing, thus, more architectural flexibility. This design philosophy has 9 conducted to ever more slender structural elements, with ever lower natural frequencies that 10 are, therefore, closer to the frequency bands of dynamic excitations associated to human 11 activities, such as walking. Within this context, this paper studies the behaviour of the 12 following flooring systems: (a) reinforced concrete slabs supported by steel beams, and (b) 13 steel floor plates supported by steel beams. The evaluation of the natural frequencies of the 14 structure and its responses (floor displacements and accelerations) to the walking activity were 15 analyzed by the simplified analytical method of AISC 360-10 code. The flooring systems were 16 modeled using the finite element software ANSYS 14.0? and the numerical results for natural 17 frequencies and floor accelerations were compared with those obtained by the simplified 18 procedure of the AISC 360-10 code. This way, it was possible to draw conclusions about the 19 dynamic behaviour of the analyzed flooring systems. 20

21

22 Index terms—flooring systems, walking loading, dynamic behaviour, structural vibrations.

#### 23 1 Introduction

rchitectural new tendencies along with current market demands are taking engineering design towards the use of flooring systems which can span great distances with a minimum number of pillars allowing, thus, more architectural flexibility. This design philosophy has conducted to ever more slender structural elements, with ever lower natural frequencies that are, therefore, closer to the frequency bands of dynamic excitations associated to human activities, such as walking.

Within this context, studies about the dynamic behaviour of commonly employed flooring systems become necessary for the evaluation of the service conditions of buildings subject to vibrations caused by human activities such as walking.

Brazilian Standard NBR 8800:2008 covers this topic very superficially. Annex L points restrictions only on the natural frequency of the floor, furnishing a simplified evaluation that depends on the total vertical displacement of the floor. However, the standard signals that this evaluation might not be adequate for the problem and leaves its application to the discretion of the engineer.

The CEB (1991) standard treats the subject in a broader manner, furnishing graphical and analytical representations for human activities, pointing some factors that influence the damping of a structure, the effects caused by vibrations on people and on the structure, tolerable acceleration values, corrective measures, and more. It is interesting to point out that the manual presents two simplified design rules. The first, High Tuning Method, being simple and efficient, limits the fundamental frequency of the floor with respect to its damping rate. This method, though, can be quite conservative. And the second, Method of Heel Impact, presents simple procedures for the computation of the frequency and initial peak acceleration of the floor.

The AISC 360-10 standard, through Murray et al. ??2003) Design Guide, also treats the subject and presents simple analytical tools for the verification of flooring systems subjected to vibrations. In a similar way to the <sup>45</sup> method described by CEB, firstly the frequency is calculated, followed by the peak acceleration. The standard

<sup>46</sup> not only presents a broader application as compared to the most commonly used methods, but also has its own

criterion based on the dynamic response of flooring systems supported by steel beams subject to walking loading,
consisting, therefore, the utilized tool in this study.

Following this directive, Mello (2005) developed numerical analyses in flooring systems subject to human activities and compared the obtained results with the simplified calculation method of AISC 360-10. Pretti (2012) performed a study about the different simplified procedures for determining peak acceleration of flooring

52 systems subject to human activities and applied these procedures in numerical examples.

## 53 **2** II.

## 54 3 SIMPLIFIED CALCULATION METHOD

55 The Steel Design Guide by ??urray et al. (2003) presents, in accordance to American Standard AISC 360-10, a 56 simplified analytical method for determining the frequency and the acceleration of a flooring system.

The following described method is applied to floor panels subject to the human activity of walking, a) Floor Natural Frequency

The most important parameter for the verification of the service limit state of excessive vibration in flooring panel systems is the natural frequency. Following, a simplified procedure for the determination of the vertical fundamental frequency of the flooring system is presented.

A simply supported beam, with uniform distributed load, has its natural frequency given by Eq. (1). $\partial$  ??" $\partial$  ??"

63 ?? = 0.18?  $\eth$  ??" $\eth$  ??"? ,(1)

64 where:

 $\delta ??"\delta ??"? = natural fundamental frequency of the beam [Hz];$ 

#### 66 4 ð??"ð??"

 $_{67}$  = gravity acceleration, 9.81 m/s 2; ? = beam deflection.

The deflection of a simply supported beam subject to uniformly distributed loading is calculated by Eq. (2).?= 5???? 4 384?? ?? ?? ?? ,(2)

where: ?? = uniformly distributed weight per unit length (actual, not design);

#### 71 5 ??

## $_{72}$ 6 = element length;

73 ?? ?? = steel elasticity modulus, 200000 MPa;

?? ?? ?? = moment of inertia of the transformed section.

A flooring system is composed by slabs supported in beams which are supported by girders, these later supported by columns. The natural frequency of the flooring system is estimated as a function of the frequency of the beam panel and the frequency of the girder panel, combining both. For this, Dunkerley's relation is used to derive the combined mode, Eq. (3):1  $\partial$  ??" $\partial$  ??" ?? 2 = 1  $\partial$  ??" $\partial$  ??" ?? 2 + 1  $\partial$  ??" $\partial$  ??"  $\partial$  ??" $\partial$  ??" 2 ,(3)

- 79 where:
- $\delta ??"\delta ??" ?? = beam panel mode frequency;$
- $\delta$  ??" $\delta$  ??" $\delta$  ??" $\delta$  ??" $\delta$  ??" = girder panel mode frequency.

The combined mode or system frequency can be estimated using Dunkerley relationship, given by Eq. (4): $\delta$  ??" $\delta$  ??" ?? = 0.18 ?  $\delta$  ??" $\delta$  ??" (? ?? + ?  $\delta$  ??" $\delta$  ??"),(4)

84 where: ? ?? = deflection of the beam panel;

? ð ??"ð ??" = deflection of the girder panel; b) Floor Acceleration Most vibration problems caused by human 85 activities involve periodic loadings, with respect to time, though walking presents itself a bit more complicated 86 since the location of the forces also varies with time. In general, a periodic loading can be represented by a 87 combination of sinusoidal forces with frequencies that are multiple or harmonics of a basic frequency, named step 88 frequency, for human activities. The load can then be represented by a Fourier series given by Eq. (5). This 89 project criterion uses as loading a single time dependent harmonic component with a frequency equal to the 90 floor fundamental frequency, according to Eq. (6). Only one component of Eq. (5) is used since all the other 91 harmonic vibrations are low when compared with the resonance associated harmonic.??(??) = ???1 + ????? 92 93 

The response function in resonance is given by Eq. (7).?? ?? ð ??"ð ??" = ?????? ?? ????? cos(2??ð ??"ð ??" 95 ?? ??),(7)

96 where:

?? ?? /ð ??"ð ??" = ratio between estimated peak acceleration and gravity acceleration; ?? = reduction
factor; ?? = floor system damping rate; ?? = effective floor weight. The reduction factor ?? takes into account
the fact that stationary resonant movement is not achieved during the walk and that the person walking and the
perturbed person are not simultaneously on the maximum displacement location. It is recommended to use ??
equal to 0.7 for walkways and 0.5 for flooring systems. The effective weight of the floor will be estimated on the

next subsection. Equation (7) can be simplified by using an approximated function for the dynamic coefficient as a function of the frequency, Eq. (8).?? = 0.83 exp(?0.353 ??") (8)

Finally, a flooring system will be considered satisfactory if it obeys Eq. (9).?? ?? ð ??"ð ??" = ?? 0 ?????? (7)

- where: ??  $0/\delta$  ??" $\delta$  ??" = limit acceleration as a function of the type of building;
- 107 The limit values ?? 0, ??, and ?? 0  $/\delta$  ??" $\delta$  ??", recommended for general occupation are given by Table 1.

#### $_{108}$ 7 c) Effective panel weight

In general, effective weights for the beam and girder panel modes can be calculated by Eq. (10).?? =  $\delta$  ??" $\delta$  ??"???,(8) where:

- 111  $\delta$  ??" $\delta$  ??" = weight per unit area; ?? = memberspan;  $\delta$  ??" $\delta$  ??" = effective width.
- For the beam, the effective width is defined as: $\partial ??"\partial ??" ?? = ?? ?? ?? ?? ?? ?? ?? ?? ?? ?? (9)$

This value is not allowed to be greater than 2/3 of the floor width, ??  $\delta$  ??" $\delta$  ??" , 1 where: For the girder, the effective width is defined as:?? ?? =  $2.\delta$  ??" $\delta$  ??" $\delta$  ??" $\delta$  ??" $\delta$  ??" $\delta$  ??" $\delta$  ??" ? ?? ?? ??  $\delta$  ??" $\delta$  ??" $\delta$  ??" ? 1 4 ?

115 ?? ð ??"ð ??" . (**10**)

This value is not allowed to be larger than 2/3 of the floor length, ?? ?? , 2 where: When the beams are continuous over their supports and with an adjacent span is greater than 0.7 times the span under consideration, the effective panel weight, ?? ?? or ??  $\partial$  ??" $\partial$  ??", can be increased by 50%. This liberalization can also be applied to rolled steel beams connected (by shear) to the web of the girder, but not to trusses connected only at their top chord.??  $\partial$  ??" $\partial$  ??" = 1.

For the combined mode, the equivalent panel weight is approximated using Eq. (??3).?? =  $\hat{1}$ ?" ??  $\hat{1}$ ?" ?? +  $\hat{1}$ ?"  $\hat{0}$  ??" $\hat{0}$ ??"  $\hat{0}$ ??" $\hat{0}$ ??"  $\hat{0}$ ??"

123 where:

124  $\hat{I}$ ?" ?? and  $\hat{I}$ ?" g = aximum deflexions of the beam and girder, respectively, due to the supported load; W j 125 and ??  $\delta$  ??" $\delta$  ??" = fective panel weights for the beam and girders panels, respectively.

If the girder span is less than the width of the beam panel,??  $\delta$  ??" $\delta$  ??" $\delta$  ??" $\delta$  ??"???? , the girder deflection,  $\hat{1}$ ?"  $\delta$  ??" $\delta$  ??" $\delta$  ??", used in Eq. (??3) is reduced to: When the edge member is a beam, the practical solution is to stiffen the edge, either by the addition of another beam or by the substitution of this member by another one that should have a 50% higher moment of inertia. If edge beam is not stiffened, its verification should be made using ?? ?? = 1.0 on Eq. (11). $\hat{1}$ ?"  $\delta$  ??"  $\delta$  ??" $\delta$  ??"  $\delta$  ??" $\delta$  ??"  $\delta$  ??" $\delta$  ??"  $\delta$  ?

When the edge member is a girder, the verification should be made according to the described procedure, except by the fact that the effective width ( $\eth$  ??" $\eth$  ??" $\eth$  ??" $\circlearrowright$  ??") should be taken equal to 2/3 of the secondary supported beam span.

The experience has shown that external floors edges of buildings do not require special attention such as the internal floor edges. The reason for this is the stiffening due to the external cladding and walkways which in general are not adjacent to external walls.

#### 137 **8 III.**

#### **138 9 SIMPLIFIED CALCULATION METHOD**

a) About the used Software For the numerical analysis was used the ANSYS 14.0?. This software is quite rich with
respect to the element library, the possible types of structural analysis, and the available numerical resources.
Besides that, ANSYS 14.0?has been well utilized by the scientific community in numerical simulations to analyze
the dynamic behaviour of structures.

#### <sup>143</sup> 10 b) Structural Model

The structural models analyzed in this paper represent an internal floor compartment, constituted by slabs or steel floor plates supported by beams that are supported by girders or columns, as in Fig. ??. The columns are not modeled. The dynamic load, ??(??), due to walking activity, is applied only at the center of the model. The beams and girders were modeled using the element BEAM 188 3-D, that has six degrees of freedom at each node, three translations and three rotations.

There is yet the seventh degree of freedom, the cross section warping, which is optional and was not used in this work. The cross section mesh for this element has a refinement option that can vary from 0 to 5. For the present work was adopted a refinement value of 2.

The slabs and the steel floor plates were modeled using the SHELL 181 element, consisting of 4 nodes and having 6 degrees of freedom per node, three translations and three rotations. This element is adequate for the analysis of thin shells or moderately thick shells, simulating both the flexure and the membrane effects.

Both elements might be displaced with respect to their geometric axis assuring, therefore, the correct placement of the slabs and the floor plates with respect to the beams.

The mesh dimensions were defined from free vibration analysis of a flooring system similar to model I, described later, with beams span of 9.0m. Analyzing Table 2, it is observed that in meshes with dimensions smaller than 0.30m, the natural frequencies present small percent deviations when compared to the 0.05m mesh. Thus, a square mesh of 0.25m side was adopted, since it gives an exact number of divisions for the models dimensions. Two types of floors were modeled, according to the following description.

#### <sup>162</sup> 11 c) Structural Model I

This model is constituted by a plan concrete slab with characteristic strength ð ??"ð ??" ???? equal to 30MPa, 163 164 specific mass of 2500kg/m 3, Poisson's ratio 0.2, and 0.15m thickness. Steel girder sections W530x74.0 of length 165 equal to 9.0m, and steel beam sections W460x52.0, equally spaced to each other at a 3.0m distance, as in Fig. 2. The beam span, ?? ?? , was varied from 6.0 to 10.5m in intervals of 0.5m. It was chosen to represent an 166 office floor, and thus a damping rate of 3% was adopted, according to Table 1. The loading to be applied is not 167 the design load as described by the procedure in section II. It was used, then, as variable loading, 0.70 kN/m 2 168 , of which 0.50 kN/m 2 accounts for the use and occupation and 0.20 kN/m 2 for the mechanical equipment and 169 covering. 170

#### $_{171}$ 12 d) Structural Model II

This model is constituted by 8mm steel plates supported by beam sections W310x28.3. Steel girder sections W460x60.0 have length equal to 9.0m, as in Fig. ??. The beam span, ?? ?? , was varied from 1.5 to 10.5m in intervals of 1.0m. The support of the floor plates in transversal directionregarding to beams were not considered in the model since they have a lesser influence on the dynamic behavior, given their reduced mass and stiffness. In this way, models with higher number of nodes and elements were avoided.

#### 177 13 Fig. 3 : Isometric view -Structural model II

This model was also conceived to represent an office floor, and thus the same damping rate of 3% was adopted. The variable loading acting adopted was the same value applied to model I, i.e., 0.70kN/m 2. The objective of

this model was to evaluate the dynamic behaviour of a flooring system composed solely of steel elements.

#### <sup>181</sup> 14 e) Loading Parameters

186 (13) f) Structur Alanalysis

The dynamic analysis performed in ANSYS 14.0? was linear elastic, without the consideration of initial imperfections. The software allows dynamic equilibrium equations resolution both by direct integration or by the modal superposition method. The direct integration method was the chosen option.

The time interval used in the integration was of 0.005s and the algorithm for the integration of the equations chosen in ANSYS 14.0? was Newmark's linear acceleration. For this algorithm, according to Clough and Penzien (1995), a time interval Î?" less or equal 10% of the excitation period yields trustworthy results. Furthermore, Bathe ??1996) shows that precise values are obtained with a time interval of approximately 1% of the excitation period. ANSYS 14.0? adopts a damping matrix proportional to the mass and stiffness matrices. For this, Year 2014 E

#### 196 15 RESULTS

The analyzed results are the natural frequencies and the accelerations of the floors subjected to Eq. (??5) dynamic loading. Several simulations were carried out for both structural models, varying, for each simulation, the beam length, L j. . a) Structural Model I

The fundamental frequencies for this model are listed in Table 3. For the computation of the ?? and ?? constants were utilized the frequencies  $\delta$  ??" $\delta$  ??" 01 and  $\delta$  ??" $\delta$  ??" 03, as shown in Table 4. With transient vibration analysis, the response for displacement and acceleration of the floor central node is obtained. Figures **??** and 5 show, respectively, the history of displacement and acceleration of model I with beams of length equal to 8.0m, i.e., ?? ?? =8.0m.? ?? ?? = 2??  $\delta$  ??" $\delta$  ??" ?? +  $\delta$  ??" $\delta$  ??" ?? ?  $\delta$  ??" $\delta$  ??" ?? ?? ?? ?? ?? ?? ?? (1?,(14) where: ?? = damping rate;

the user must provide the values of the constants ?? and ?? , that are calculated according to Eq. (??6):

Fig. ?? : Vertical displacement for the computational model with ?? ?? =8.0m

Fig. ?? : Vertical acceleration for the computational model with ?? ?? =8.0m

At Table 5 are displayed the fundamental frequency results obtained by ANSYS 14.0? and by the simplified procedure of the AISC 360:10 code.

Comparing these results, a slight percent difference can be noticed. Table 6 presents the results for peak acceleration obtained by ANSYS 14.0? and by the simplified procedure of the AISC 360:10 code. The ratio between the time interval and the period,  $\hat{1}$ ?"/, utilized in the models is within the interval from 0.0221 to 0.0372.

214 (1) Reduction factor, ??.

(2) The value for limit acceleration was calculated for  $\delta$  ??" $\delta$  ??"=9.81m/s 2 . This value is also suggested by CEB ??1991).

The plots at Figures 6 and 7 show, respectively, the variation of the fundamental frequency and the peak acceleration with the value of the beam span. Table 10 presents the results for peak acceleration obtained by ANSYS 14.0? and by the simplified procedure of the AISC 360:10 code. The ratio between the time interval and the period,  $\hat{1}$ ?" ?? /??, utilized in the models is within the interval from 0.0207 to 0.0668.The plot at Fig. 9 shows the variation of the peak acceleration with the value of the secondary beam span. (2) The value for limit acceleration was calculated for  $\delta$  ??" $\partial$  ??"=9.81m/s 2. This value is also suggested by CEB (1991).

## 223 16 CONCLUSION

Two representative models of flooring systems were simulated in the ANSYS 14.0? finite element software to study their dynamic behaviour due to loadings caused by the human activity of walking. Both models simulate an inner compartment of an office floor, their difference being that model I is composed by concrete slabs and steel beams and model II is composed by floor plates and steel beams. The numerical results were confronted with the analytical formulation of the AISC 360:10 code.

When analyzing the percent deviation of Tables 5 and 9, it is concluded that this value increases as the span length ?? ?? increases. Probably, this is due to the fact that Eq. (4) of the AISC code calculates the system frequency from the static displacements of the beams and girders, assuming a simplified uniformly distributed load for both beams. At the ANSYS 14.0? models, these simplifications were not made.

Model I presents accelerations greater than the allowed limit when the span length of the beam exceeds 9m. Therefore, vibration analysis becomes an important factor for the type of occupation studied and should be taken into account in the design of the building floor.

With respect to model II, the floor presents peak acceleration values higher than the limit for small spans of the beam, above 2.5m. This shows that the flooring system is far more susceptible to vibrations than the model I system, and does not configure a good structural solution for the studied type of occupation.

Still in relation to the obtained results of accelerations, plots at Fig. 7 and Fig. ??0, the same behaviour which occurred for the fundamental frequencies can be observed: increase of the differences between the results calculated by the finite element software and by the AISC code with the increase of the beam span ?? ?? . This probably occurs due to the conditions imposed on the computation of the effective widths ð ??"ð ??" ð ??"ð ??" ?? and ð ??"ð ??" ð ??"ð ??" , Eq. (11) and (12), respectively, that imply on the significant reduction of the floor effective weight ??, giving higher values for the acceleration according to Eq. (9). In general, it can be concluded that:

246 ? The fundamental frequencies calculated by the analytical method of the AISC 360-10 code are, most 247 frequently, slightly smaller than numerical fundamental frequencies, configuring a trustworthy analytical 248 simplification for the determination of the floor fundamental frequency;

249 ? Regarding the accelerations, the values obtained by the analytical method were found consistently larger 250 than those by the computational model, showing that the AISC 360-10 formulation presents a reasonable safety 251 margin.

252 VI. <sup>1 2 3</sup>

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<sup>&</sup>lt;sup>2</sup>The floor length must be multiplied by 3 when dealing with a typical internal room. Vibrations in Steel-Frame Floors due to Human Activities © 2014 Global Journals Inc. (US)

 $<sup>{}^{3}</sup>$ Year 2014 E



Figure 1:



Figure 2:



Figure 3: Figure 2 :



Figure 4: Fig. 2 :

### 1

Purpose	Constant force P 0	Damping Rate ?	Limit acceleration a 0 /g.100%
Offices, residences and	0.29 kN	$0.02 - 0.05^*$	0.5%
churches			
Shopping centers	0.29 kN	0.02	1.5%
Footbridge-Indoor	0.41 kN	0.01	1.5%
Footbridge-Outdoor	0.41 kN	0.01	5.0%
0.02			

[Note: Source: Murray et al. (2003)]

Figure 5: Table 1 :

#### $\mathbf{2}$

# Mesh [m]

f n [Hz]

Percent Deviation

Figure 6: Table 2 :

3										
$\mathbf{L}$	[m]			Fundame	ntalFrequen	cies[Hz]				
j										
	f 01	f 02	f 03	f 04	f $05$	f 06	f 07	f 08	f 09	f 10
10.5	4.43	6.85	9.61	10.18	16.92	18.78	19.57	24.38	29.64	32.41
10.0	4.71	7.42	9.92	10.77	17.82	19.30	20.83	25.81	31.58	32.79
9.5	5.00	8.07	10.23	11.45	18.85	19.91	22.20	27.32	33.25	33.79
9.0	5.31	8.80	10.56	12.26	20.02	20.64	23.69	28.94	33.81	36.29
8.5	5.63	9.63	10.91	13.21	21.38	21.52	25.31	30.68	34.50	39.11
8.0	5.96	10.58	11.28	14.33	22.59	22.97	27.07	32.59	35.36	42.28
7.5	6.31	11.65	11.67	15.65	23.90	24.83	28.98	34.72	36.47	45.82
7.0	6.67	12.08	12.88	17.20	25.51	27.03	31.08	37.16	37.90	49.53
6.5	7.05	12.54	14.29	18.99	27.50	29.64	33.37	39.82	40.01	53.24
6.0	7.45	13.03	15.90	21.06	29.98	32.76	35.88	42.41	43.40	57.50

Figure 7: Table 3 :

4					
Lj [m]	f 01 [Hz]	f 03 [Hz]	$0.83 \exp(-0.35.f$ 01)	?	??
10.5	4.43	9.61	0.1761	$0.03\ 1.143$	0.000680
10.0	4.71	9.92	0.1597	$0.03\ 1.204$	0.000653
9.5	5.00	10.23	0.1441	$0.03\ 1.267$	0.000627
9.0	5.31	10.56	0.1294	$0.03 \ 1.332$	0.000602
8.5	5.63	10.91	0.1156	$0.03\ 1.400$	0.000577
8.0	5.96	11.28	0.1029	$0.03 \ 1.471$	0.000554
7.5	6.31	11.67	0.0911	$0.03\ 1.544$	0.000531
7.0	6.67	12.88	0.0803	$0.03\ 1.657$	0.000488
6.5	7.05	14.29	0.0704	$0.03\ 1.780$	0.000448
6.0	7.45	15.90	0.0613	$0.03 \ 1.912$	0.000409



 $\mathbf{5}$ 

	8 0F-05						
	6.0E-05						
	4.0E-05						
Displacen	4.02-05						
[m]							
[111]	-4 0E-05						
	-6.0E-05						
	-8.0E-05						
	-8.02-05	1.0				6.0	70.8
	0.0	1.0			Time [a]	0.0	1.0 8.
	0.1				rime [s]		
	0.1						
	0.08						
A 1 4							
Accelerati	$1040.04 - 0.02 \ 0 \ 0.02 \ 0.04$						
$[m/s^2]$	0.00						
	-0.06						
	-0.08						
	-0.1						
	0.0	1.0	2.0	3.0	4.0	5.06.0	7.0 8.
					Time [s]		
				: Natural frequ	lencies		
		L j	Comp	utational Model	AISC	Percent	
		[m]				Deviation	
		10.5		4.43	4.06	8.33%	
		10.0		4.71	4.34	7.84%	
		9.5		5.00	4.65	7.07%	

Figure 9: Table 5

## 6

Lj [m] C	omputationalModel	0.5 (1) x Computa- cionalModel	AISC	LIMIT $0.5\%$ g (2)
10.5	0.1149	0.0574	0.0790	0.0491
10.0	0.1103	0.0552	0.0750	0.0491
9.5	0.1053	0.0527	0.0707	0.0491
9.0	0.0999	0.0499	0.0679	0.0491
8.5	0.0940	0.0470	0.0648	0.0491
8.0	0.0880	0.0440	0.0611	0.0491
7.5	0.0818	0.0409	0.0570	0.0491
7.0	0.0756	0.0378	0.0516	0.0491
6.5	0.0697	0.0348	0.0462	0.0491
6.0	0.0639	0.0320	0.0411	0.0491

Figure 10: Table 6 :

7										
${ m L}$ j $[{ m m}]$	f 01	f 02	f 03	Fundamer	ntal Frequenc	cies [Hz] f	04 f 05 f 06 f 07	f 08	f 09	f 1
10.5	4.13	5.35	5.80	6.24	6.88	7.70	8.74	9.19	9.97	11
9.5	4.68	6.39	6.92	7.38	8.01	8.82	9.73	9.84	11.03	12
8.5	5.31	7.74	8.41	8.88	9.47	10.21	10.30	11.12	12.11	13
7.5	6.00	9.53	10.41	10.85	10.90	11.36	11.92	12.55	13.15	13
6.5	6.77	11.57	11.91	12.94	13.13	13.54	13.97	14.19	14.35	14
5.5	7.61	12.32	14.46	14.55	15.06	15.37	16.20	16.79	17.01	17
4.5	8.55	13.20	15.85	16.11	17.04	18.34	18.99	19.75	20.92	21
3.5	9.66	14.27	18.59	18.98	19.94	21.39	23.13	23.53	24.91	26
2.5	11.11	15.61	24.99	25.46	26.33	27.57	28.46	29.10	30.70	31
1.5	13.36	17.13	34.92	39.09	39.30	39.63	40.06	40.33	40.64	40

Figure 11: Table 7 :

Li [m]	f 01	f 03	$0.83 \exp(-0.35 \text{ f} 01)$	?	?	?
20 []	[Hz]	[Hz]		·	·	·
10.5	4.13	5.80	0.1954	0.03	0.909	0.000962
9.5	4.68	6.92	0.1611	0.03	1.053	0.000823
8.5	5.31	8.41	0.1295	0.03	1.227	0.000696
7.5	6.00	10.41	0.1016	0.03	1.435	0.000582
6.5	6.77	11.91	0.0777	0.03	1.626	0.000511
5.5	7.61	14.46	0.0579	0.03	1.879	0.000433
4.5	8.55	15.85	0.0417	0.03	2.093	0.000391
3.5	9.66	18.59	0.0282	0.03	2.397	0.000338
2.5	11.11	24.99	0.0170	0.03	2.900	0.000265
1.5	13.36	34.92	0.0077	0.03	3.642	0.000198
	1 . 1	C 1	. 1		1 / 1 /1	· · · ·

At Table 9 are displayed the fundamental

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frequency results obtained by ANSYS 14.0? and by the fundar simplified procedure of the AISC 360:10 code. beam.

be noticed. Fig. 8 plot shows the variation of fundamental frequency plotted against the span of beam

Comparing these results, a slight percent difference can

Table 9 : Natural frequencies		
Computational Model	AISC	Percent
		Deviation
4.13	4.04	2.24%
4.68	4.57	2.44%
5.31	5.18	2.41%
6.00	5.86	2.36%
6.77	6.61	2.31%
7.61	7.45	2.05%
8.55	8.41	1.61%
9.66	9.57	0.94%
11.11	11.08	0.31%
13.36	13.40	-0.32%
	Table 9 : Natural frequencies Computational Model 4.13 4.68 5.31 6.00 6.77 7.61 8.55 9.66 11.11 13.36	Table 9 : Natural frequencies Computational ModelAISC $4.13$ $4.04$ $4.68$ $4.57$ $5.31$ $5.18$ $6.00$ $5.86$ $6.77$ $6.61$ $7.61$ $7.45$ $8.55$ $8.41$ $9.66$ $9.57$ $11.11$ $11.08$ $13.36$ $13.40$

Figure 12: Table 8 :

## 10

Lj [m]	Computational	$0.5(1) \ge 0.5$	AISC	LIMIT	0.5%g
	Model	tional Model		(2)	
10.5	0.4751	0.2376	0.3722	0.0491	
9.5	0.4384	0.2192	0.3124	0.0491	
8.5	0.3765	0.1882	0.2564	0.0491	
7.5	0.3115	0.1558	0.2073	0.0491	
6.5	0.2538	0.1269	0.1664	0.0491	
5.5	0.2050	0.1025	0.1337	0.0491	
4.5	0.1653	0.0826	0.1074	0.0491	
3.5	0.1298	0.0649	0.0853	0.0491	
2.5	0.0974	0.0487	0.0641	0.0491	
1.5	0.0605	0.0302	0.0408	0.0491	
(1)					
Reduction					
factor, ??.					

Figure 13: Table 10 :

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