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Bending Moments in Beams of Two Way Slab Systems

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Keywords: two way slab, sap2000, direct design method, beams.

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Bending Moments in Beams of Two Way Slab Systems

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I. Introduction and Background

There are many methods that are used for analysis and design of two way slab systems. The accuracy of these methods depends on the assumptions that are stated in the method procedure. The most common method is the ACI direct design method. This method depends on determining the moments in the frames using coefficients. Then, the frame moments are distributed to the beam and the slab using tables. These tables depend on three main variables which are: type of moment; positive or negative, width of frame/ span length ratio and beam/slab flexural stiffness ratio. Really, these variables or factors are very important in distribution of moments in the beam and in the slab.

II. Methodology

Two way solid slabs with beams, having different beam/slab flexural stiffness ratios will be considered in this study. These slabs shall have different long span, L/short span, B ratios. This study will include four slabs with L/B of 1, 1.25, 1.5 and 2. Each slab shall have different values of beam/slab flexural stiffness ratios.

The slabs will be subjected to distributed gravity load of 15kN/m2 including weights of beams and slab. This load will be applied as an area load on the slab in sap2000 structural model and the self-weight of the structure is considered to be equal to zero.

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The four slabs shall have panels' spans of 8x8m, 8x6.4m, 8x5.33m and 8x4m. The small spans will be in x- direction (horizontal axis) and the long spans will be in y- direction (vertical axis). The slab thickness is 0.22m for all slabs. The beams in the two directions will have same dimensions. The beam sizes are 0.40m width and with variable depth that varies from 0.3m to 1.0m with step of 0.10m. The modifiers for effective moment of inertia will not be used. The supporting columns have a square cross section of 0.6m side length.

The slab system is modeled as three dimensional building structure that has a column height of 3.5m with columns below and above. The concrete modulus of elasticity, E is equal to 2.5×10^4MPa.

Two perpendicular interior frames in each slab shall be studied. Frame A will be in y- direction with longer spans and frame B will be in x- direction with shorter spans. The moments in the beams of these two frames in each slab will be recorded to study the relation between the beam/slab stiffness ratio and the moment distribution to beams. In addition, these two frames shall be analyzed as plane frames with loads equal to frame width multiplied by the slab unit load and with triangular or trapezoidal load shape depends on the frame width, as stated by the 45 degrees- load- distribution principle, to study and compare the bending moments in these frames with that in the three dimensional models.

III. Three Dimensional Structural Analysis of Slabs

The computer program sap2000 is used for structural modeling of the slab systems. The slab is modeled as area elements while beams and columns are modeled as frame or line elements. The slab is subjected to uniformly distributed load of 15kN/m2. Using sap2000, the moments in beams will be determined. The beams are part of the frame column strip. The column strip stiffness is larger than middle strip stiffness, so the moments in the column strip are larger than the moments in the middle strip. The moment ratios for column strip depend on the aspect ratio of panels. The moment ratios for beams depend on their flexural stiffness. As the flexural stiffness of the beam increases the moments that will carry will increase show in tables 1 to 4. These tables illustrate the moment values in long direction interior beams and in short direction interior beams. The moments in the exterior span and in the interior span are stated in these tables. The main moments in beams are exterior negative moment, interior negative moment in exterior span, interior negative moment in interior span and positive moments in exterior and interior spans. All moment values are in kn.m.

Table 1 : Bending moments in beams for slab 8x8m from three dimensional structural analysis

Beam size, width x depth(mm)	Direction	Exterior span			Interior span		
		Exterior negative moment	Positive moment	Interior negative moment	Interior negative moment	Positive moment	Interior negative moment
400x300	**Long, short**	119.4	54.1	110.9	92.3	44.5	92.3
400x400	**Long, short**	174.8	101	181	157	84.9	157
400x500	**Long, short**	210	148.6	241.4	212.2	124.4	212.2
400x600	**Long, short**	223.6	190.5	288.3	253.8	155.6	253.8
400x700	**Long, short**	219.7	225.7	323.6	284.3	177.1	284.3
400x800	**Long, short**	203.2	257.7	345	307.1	190.1	307.1
400x900	**Long, short**	178.4	285.9	369.6	325.1	196.9	325.1
400x1000	**Long, short**	148.8	312.2	384.1	340	199.2	340

Table 2 : Bending moments in beams for slab 8x6.4m from three dimensional structural analysis

Beam size, width x depth(mm)	irection	Exterior span			Interior span		
		Exterior negative moment	Positive moment	Interior negative moment	Interior negative moment	Positive moment	Interior negative moment
400x300	**Long**	100.5	46.5	92	76.2	38	76.2
	Short	72.16	39.2	72	62.2	32.9	62.2
400x400	**Long**	148.4	86.5	151.6	131.9	73.2	131.9
	Short	102	69.5	113.5	100.5	58.9	100.5
400x500	**Long**	179.8	128	205	181.8	108.7	181.8
	Short	116.9	97.9	145.8	129.2	80.8	129.2
400x600	**Long**	193.8	165	248.5	221.4	137.8	221.4
	Short	117.8	121.5	168.1	148	95.2	148
400x700	**Long**	193.1	196.3	282.4	251.5	158.8	251.5
	Short	108.4	141.2	183.2	160.4	103	160.4
400x800	**Long**	181.6	223.6	308.6	274.6	172.4	274.6
	Short	92.5	158.4	193.4	169.3	106.1	169.3
400x900	**Long**	162.9	248.7	328.4	292.8	180.3	292.8
	Short	72.8	174.1	200.1	176.4	106.3	176.4
400x1000	**Long**	140.1	271.6	343.2	307.6	184	307.6

Table 3 : Bending moments in beams for slab 8x5.33m from three dimensional structural analysis

Beam size, width x depth(mm)	Direction	Exterior span			Interior span		
		Exterior negative moment	Positive moment	Interior negative moment	Interior negative moment	Positive moment	Interior negative moment
400x300	**Long**	88.3	41.3	79.5	66.5	34.4	66.5
	Short	47.3	29.8	49.7	44.2	25.8	44.2
400x400	**Long**	129.8	76.7	131.8	116.2	66.1	116.2
	Short	64.1	50.6	75.7	68.3	43.7	68.3
400x500	**Long**	157.9	113	179.7	161.4	98	161.4
	Short	69.7	68.3	93.8	84	56.5	84
400x600	**Long**	171.2	145.4	219.3	197.8	124.3	197.8
	Short	65.9	82.3	104.9	92.8	63.2	92.8
400x700	**Long**	171.9	172.9	250.6	225.8	143.4	225.8
	Short	56.1	94	111.3	97.8	65.5	97.8
400x800	**Long**	163.1	196.2	274.7	247.2	155.9	247.2

	Short	42.8	104.4	114.8	101.3	65.2	101.3
400x900	**Long**	148.2	217.9	293.1	263.9	163.2	263.9
	Short	27.8	114.3	116.4	104.2	63.5	104.2
400x1000	**Long**	129.6	237.6	306.8	277.4	166.9	277.4
	Short	12.5	123.6	116.8	106.8	61.3	106.8

Table 4 : Bending moments in beams for slab 8x4m from three dimensional structural analysis

Beam size, width x depth(mm)	**Direction**	**Exterior span**			**Interior span**		
		Exterior negative moment	**Positive moment**	**Interior negative moment**	**Interior negative moment**	**Positive moment**	**Interior negative moment**
400x300	**Long**	71.9	35.3	64.3	55.7	30.8	55.7
	Short	23.4	19.7	26.5	24.5	17.8	24.5
400x400	**Long**	105	63.9	107	97	57.3	97
	Short	28.9	30.7	37.8	35.1	27.5	35.1
400x500	**Long**	127.6	92.3	146.1	134.2	82.9	134.2
	Short	27.9	38.4	44	40.2	32.6	40.2
400x600	**Long**	138.6	117.1	178.3	163.8	103.4	163.8
	Short	22.5	43.9	46.7	41.9	34	41.9
400x700	**Long**	139.8	137.8	203.5	186.3	118	186.3
	Short	15	48.4	47.4	42.5	33.3	42.5
400x800	**Long**	133.6	155.3	222.8	203.2	127.4	203.2
	Short	6.6	52.8	46.8	42.7	31.8	42.7
400x900	**Long**	122.7	171.3	237.4	216.2	132.9	216.2
	Short	-2	57.1	45.4	43	30.1	43
400x1000	**Long**	109	186.1	248.3	226.7	135.5	226.7
	Short	-23	62.2	43.6	43.2	28.6	43.2

IV. Two Dimensional Structural Analysis of Beams

The computer program sap2000 is used for structural modeling of interior beams in the slab systems in long and in short directions. The frames that are composed of these beams and the supporting columns are analyzed as two dimensional structures. The frame has horizontal beams of three spans and top and bottom columns. The columns have a height of 3.5m. The analysis is done for beams of width 400mm and depth varies from 300 to 1000mm for the different slab systems. The load on the beam in the frame is equal to frame width multiplied by the slab area load. The loads on the slab are transferred to beams based on the 45 degrees- load- distribution principle. The shape of the span load whether it is triangle or trapezoid depends on the aspect ratio of the panel. The load shape is triangle for spans in a square panel, while the load shape is triangle for short span and trapezoid for long span in a rectangular panel. Tables 5 to 8 illustrate the moment values in the beams in the different slabs.

Table 5 : Moments in beams determined by 45 degrees- load distribution principle for two way slab 8x8m

Beam size, width x depth(mm)	**Direction**	**Exterior span**			**Interior span**		
		Exterior negative moment	**Positive moment**	**Interior negative moment**	**Interior negative moment**	**Positive moment**	**Interior negative moment**
400x300	**Long, short**	321.1	238.7	331.3	328.2	236.4	
400x400	**Long**	312	241.7	335.3	328.3	236.2	
400x500	**Long**	298.2	246.4	340.9	328.7	235.8	
400x600	**Long**	268.6	253	347.8	329.7	234.8	
400x700	**Long**	257.9	261.4	355	331.5	233.1	
400x800	**Long**	233.4	271.3	361.8	334.2	230.4	
400x900	**Long**	207.7	282.2	367.7	337.7	226.9	
400x1000	**Long**	182	293.8	372.3	341.8	222.7	

Table 6 : Moments in beams determined by 45 degrees- load distribution principle for two way slab 8x6.4m

Beam size, width x depth(mm)	Direction	Exterior span			Interior span		
		Exterior negative moment	Positive moment	Interior negative moment	Interior negative moment	Positive moment	Interior negative moment
400x300	Long	305.2	223.5	315	312	221.4	
	Short	154.5	124.1	160.8	158.9	122.9	
400x400	Long	296.6	226.3	318.8	312.1	221.2	
	Short	148.9	125.7	163.1	159	122.7	
400x500	Long	283.4	230.6	324.1	312.5	220.8	
	Short	140.6	128.3	166.3	159.3	122.4	
400x600	Long	266	236.8	330.6	313.5	219.9	
	Short	129.9	131.9	169.9	160	121.7	
400x700	Long	245	244.5	337.5	315.2	218.2	
	Short	117.3	136.4	173.5	161.2	120.6	
400x800	Long	221.7	253.7	344	317.7	215.7	
	Short	103.7	141.6	176.6	162.9	118.9	
400x900	Long	197.3	264	349.6	321	212.3	
	Short	89.8	147.4	179	164.9	116.8	
400x1000	Long	162.7	274.7	354	325	208.4	
	Short	76.4	153.3	180.6	167.2	114.5	

Table 7 : Moments in beams determined by 45 degrees- load distribution principle for two way slab 8x5.33m

Beam size, width x depth(mm)	Direction	Exterior span			Interior span		
		Exterior negative moment	Positive moment	Interior negative moment	Interior negative moment	Positive moment	Interior negative moment
400x300	Long	277.4	197.7	286.3	283.5	195.8	
	Short	85.5	71.3	87.7	86.5	70.4	
400x400	Long	269.5	200.3	289.7	283.6	195.7	
	Short	79.8	72.6	89.2	86.6	70.3	
400x500	Long	257.5	204.2	294.6	284	195.3	
	Short	74.4	74.5	91.1	86.8	70	
400x600	Long	241.6	209.8	300.5	284.9	194.4	
	Short	67.6	77.2	93.1	87.3	69.5	
400x700	Long	222.5	216.9	306.8	286.4	192.9	
	Short	59.8	80.4	95	88.2	68.7	
400x800	Long	201.2	225.3	312.7	288.7	190.6	
	Short	51.5	84	96.4	89.2	67.6	
400x900	Long	178.9	234,6	317.8	291.8	187.6	
	Short	43.4	87.9	97.4	90.5	66.3	
400x1000	Long	156.7	244.4	321.8	295.3	184	
	Short	35.6	91.8	97.9	91.8	65	

Table 8 : Moments in beams determined by 45 degrees- load distribution principle for two way slab 8x4m

Beam size, width x depth(mm)	Direction	Exterior span			Interior span		
		Exterior negative moment	Positive moment	Interior negative moment	Interior negative moment	Positive moment	Interior negative moment
400x300	Long	226.1	155.4	233.4	231.2	153.8	
	Short	30.6	30.3	32.7	32.2	29.8	
400x400	Long	219.7	157.5	236.2	231.3	153.7	
	Short	28.7	31	33.4	32.2	29.7	
400x500	Long	209.8	160.8	240.2	231.6	153.4	
	Short	26	32	34.1	32.4	29.5	

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400x600	**Long**	196.8	165.3	245.1	232.3	152.7
	Short	22.8	33.4	34.9	32.6	29.3
400x700	**Long**	181.1	171.2	250.2	233.5	151.5
	Short	19.2	35	35.4	33.1	28.9
400x800	**Long**	163.6	178	255.1	235.4	149.6
	Short	15.6	36.8	35.6	33.5	28.4
400x900	**Long**	145.4	185.7	259.3	237.9	147.1
	Short	12.2	38.6	35.6	34	27.9
400x1000	**Long**	127.1	193.8	262.5	240.9	144.1
	Short	8.9	40.4	35.4	34.5	27.4

V. Moments in Beams

The moments in beams are determined from three- dimensional analysis of slab system and from two- dimensional analysis of frame. Three dimensional structural analysis shows that the moments in the beams are increased as their flexural stiffness increases. The negative moment at the exterior support increases as the stiffness ratio increases up to about one, then the moment decreases. The stiffness ratio is given by:

$$stiffness\ ratio = \frac{I_b}{I_s}\frac{L_2}{L_1}$$

Where:

Ib= the moment of inertia of the beam

Is= the moment of inertia of the slab

L1= span length center to center of supports (columns)

L2= frame width (half distance to next column lines)

This stiffness ratio is used in ACI (American Concrete Institute) direct design method as

$$\alpha_{f1}\frac{l_2}{l_1}$$

Where l2 is the frame width, l1 is the span length and $\alpha f1$ is beam moment of inertia divided by slab moment of inertia.

Tables 9 and 10 illustrate the percentage of moments in beams related to maximum obtained moment for stiffness ratios one and two respectively. This percentage represents the beam efficiency in the frame. It is shown that the beam efficiency for the positive moment in exterior span is about 77% and 71% for long and short beams respectively at stiffness ratio of two. The beam efficiency for the positive moment in interior span is not less than 90%, while the beam efficiency for interior negative moments is not less than 85% at stiffness ratio of two.

Table 9 : Beam efficiency for stiffness ratio of one

Slab	Direction	Positive moments %		Negative moments %	
		Exterior span	Interior span	Exterior span	Interior span
8x8		61	78	75	75
8x6.4	**long**	61	75	73	72
	short	60	80	76	75
8x5.33	**long**	61	74	72	71
	short	59	96	81	81
8x4	**long**	63	76	72	72
	short	60	100	100	91

Table 10 : Beam efficiency for stiffness ratio of two

Slab	Direction	Positive moments %		Negative moments %	
		Exterior span	Interior span	Exterior span	Interior span
8x8		77	92	87	87
8x6.4	**long**	77	90	86	86
	short	75	98	90	88
8x5.33	**long**	78	90	85	85
	short	71	100	97	89
8x4	**long**	79	90	86	86
	short	82	100	100	98

The two dimensional analysis of the frame illustrates that the moments in the interior span do not much vary as the flexural stiffness of the beam increases. It shown that, the negative moments in the interior span slightly increase as their flexural stiffness increases, while the positive moment slightly decreases. This variation of moment is about ±8%. The negative moment at exterior support (column) is much varied as the flexural stiffness of the beam increases. This moment will decrease as the flexural stiffness of the beam increases. This result is expected as the support fixity increases as the beam size decreases. It is known that the positive moment and the interior negative moment in the exterior span will increase as the exterior negative moment decreases. The increase percentage in exterior negative moment as the beam flexural stiffness decreases ranges from 175% to 340%. The percentage increase is about 12.5% and less than 33% for the interior negative moment and the positive moment in the exterior span respectively.

VI. Relation between Moments in Beams Determined by Three and Two Dimensional Structural Analyses

Figure 1 shows the relation between beam moments determined by three and two dimensional analyses. This figure illustrates that at a specified value of stiffness ratio, the moments in beams determined by three dimensional structural analysis of slabs are equal to that determine by two dimensional structural analysis of frame based on the 45 degrees- load- distribution principle. Table 11 shows the stiffness ratio for equal moments.

Table 11 : stiffness ratio at which moments in beams determined by three and two dimensional structural analyses are equal

Slab	Direction	L2/L1	Stiffness ratio
8x8	**Long, short**	**1**	>3
8x6.4	**long**	**1.25**	>3
	short	**0.8**	2
8x5.33	**long**	**1.5**	>3
	short	**0.67**	1
8x4	**long**	**2**	>3
	short	**0.5**	0.5

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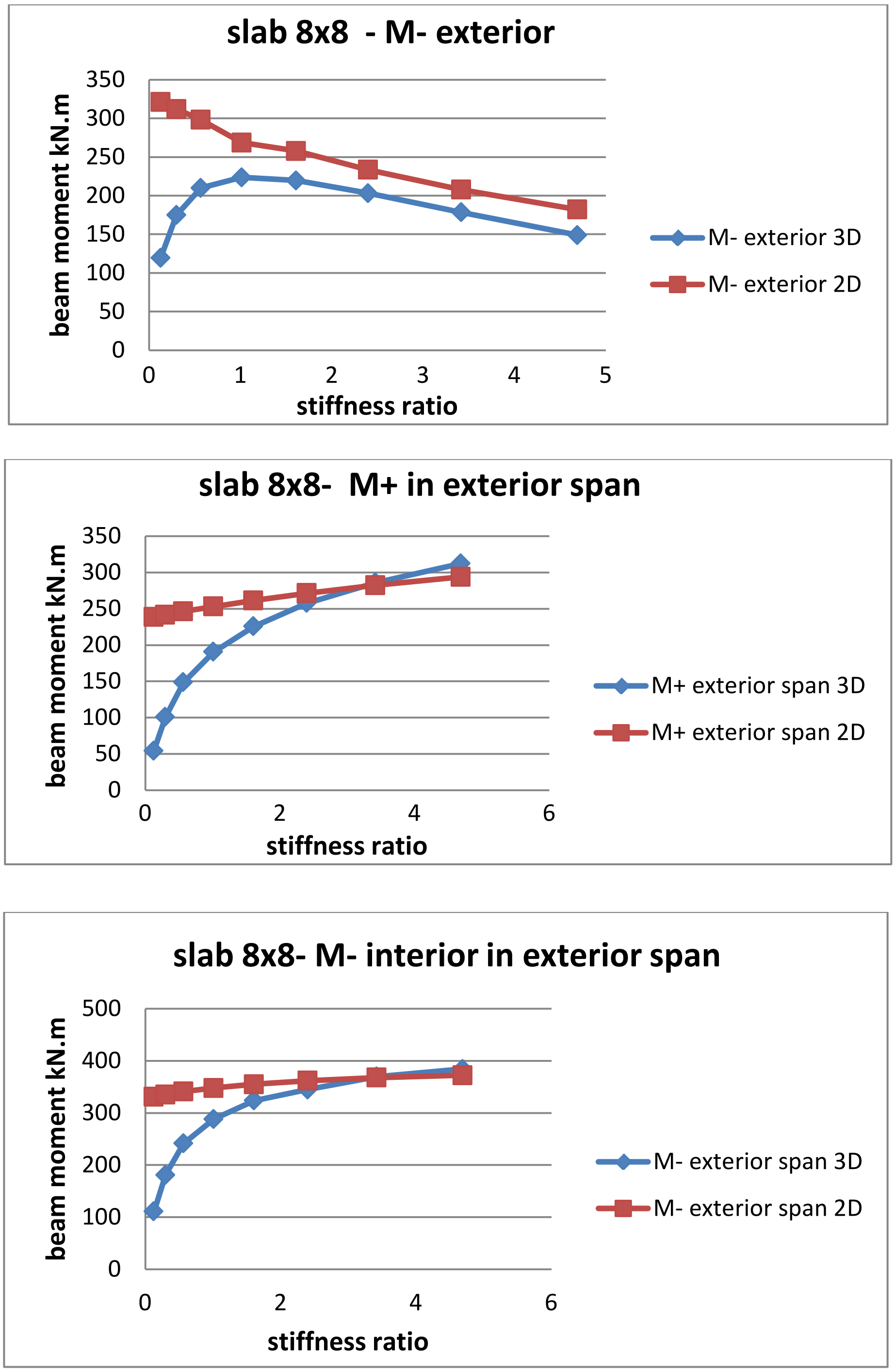
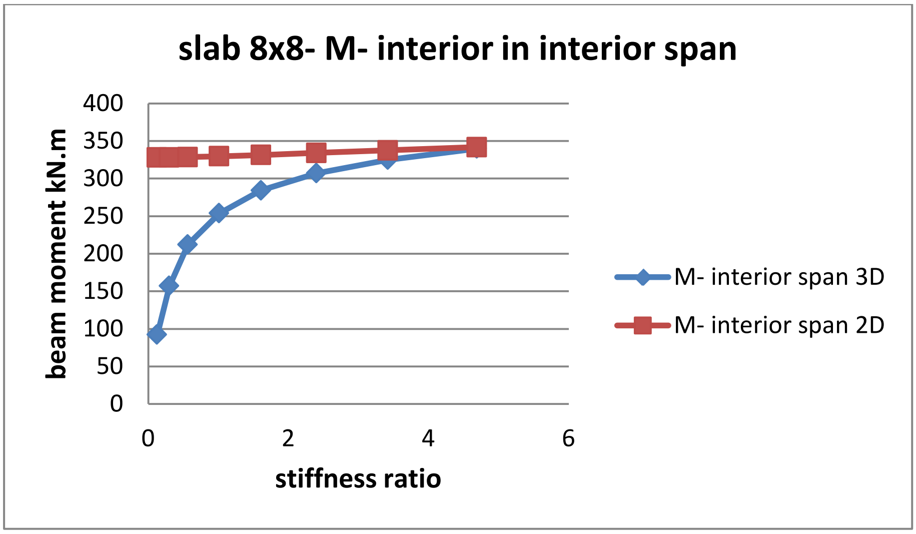
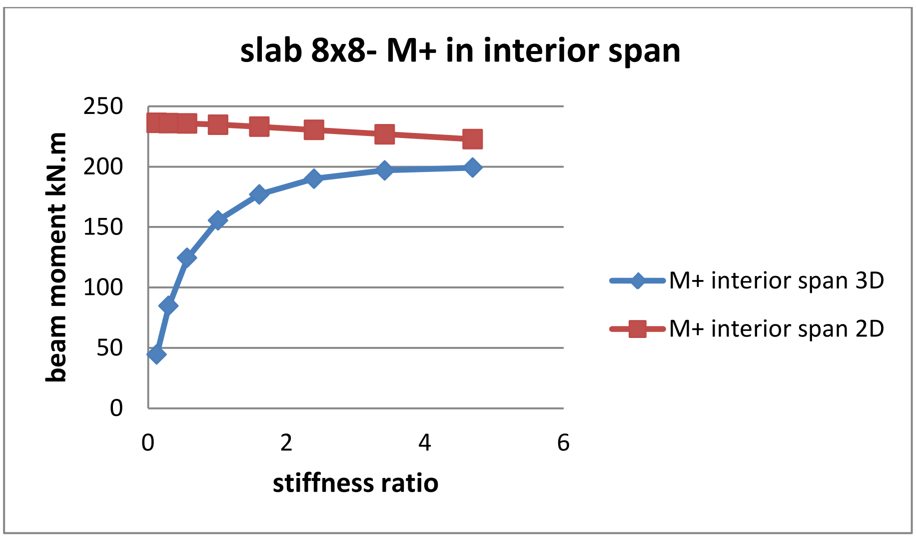


Figure 1 : Relation Between moments in beams determined by three and two dimensional analyses- A





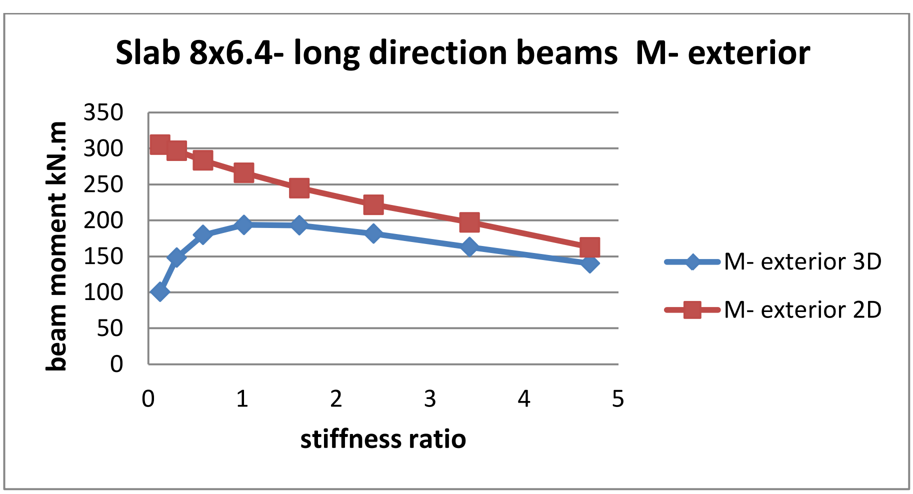


Figure 1 : Relation Between moments in beams determined by three and two dimensional analyses- B

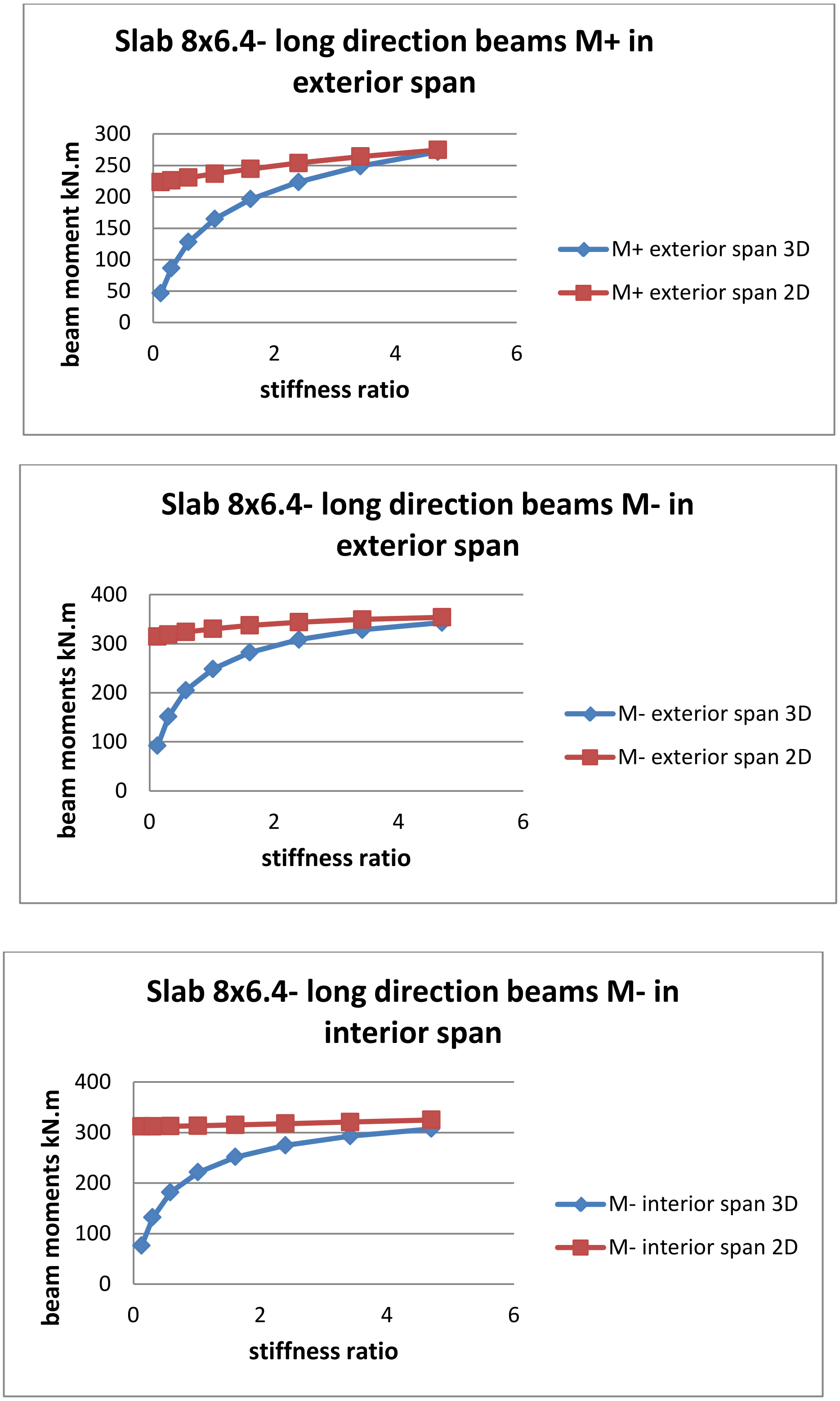
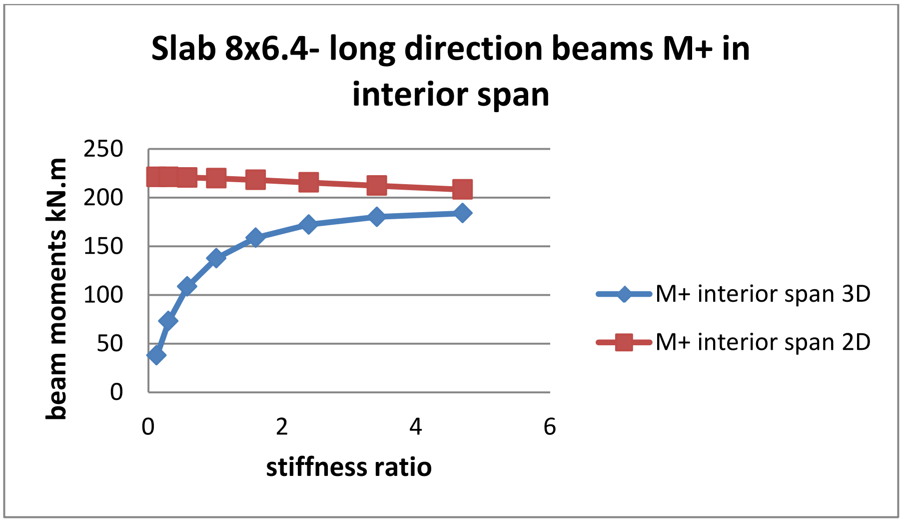
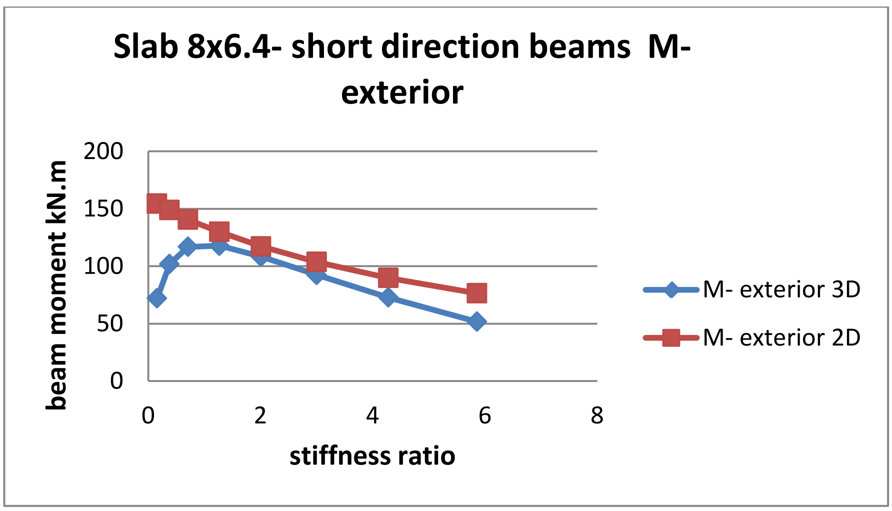


Figure 1 : Relation Between moments in beams determined by three and two dimensional analyses- C





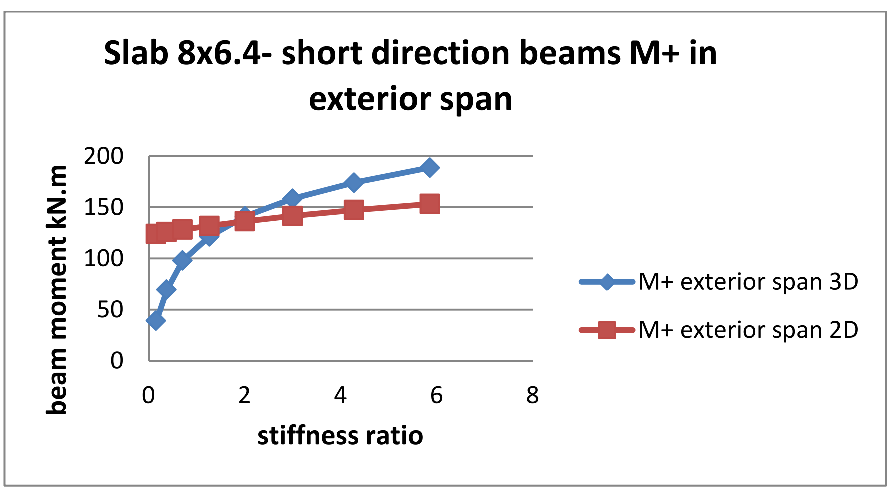


Figure 1 : Relation Between moments in beams determined by three and two dimensional analyses- D

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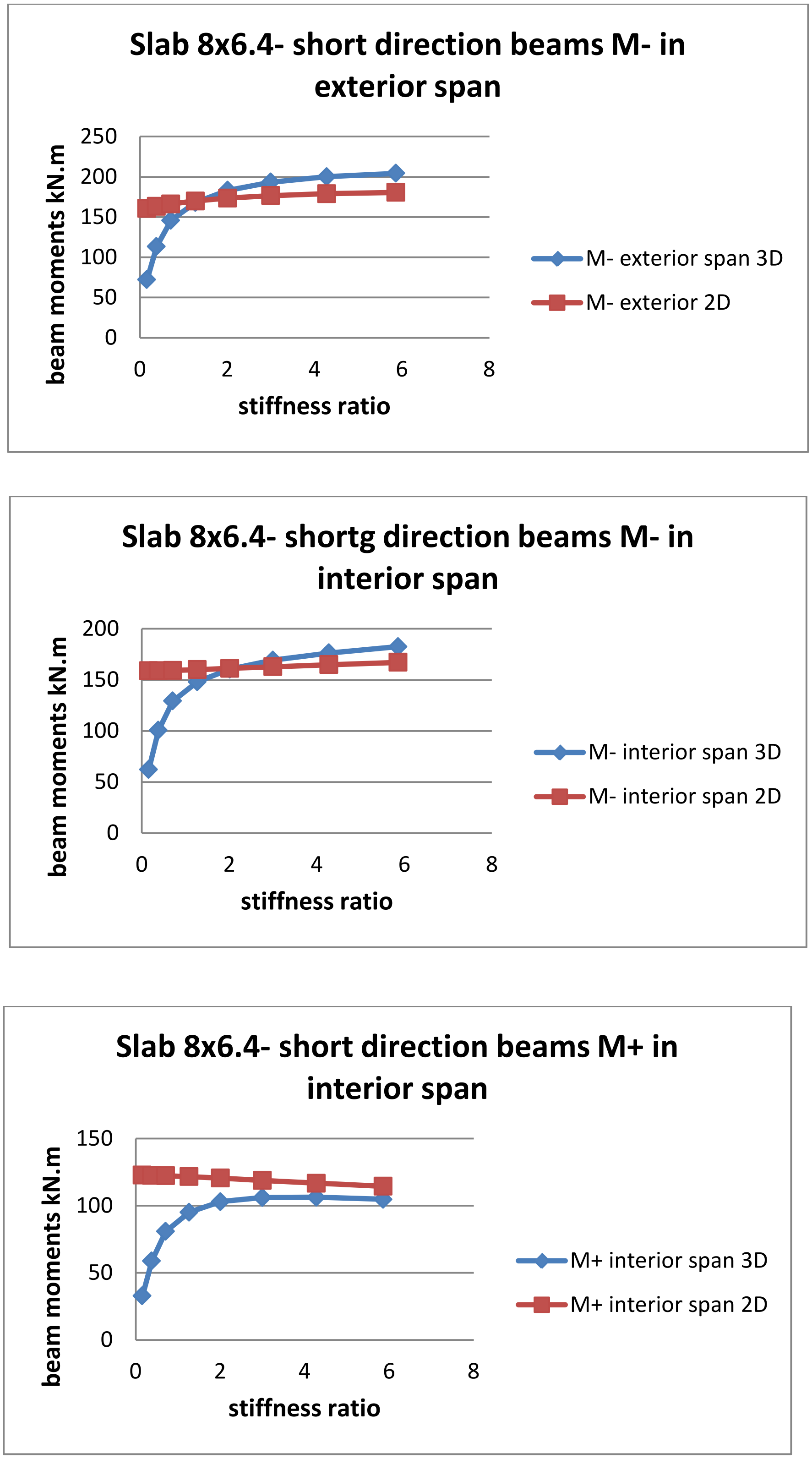
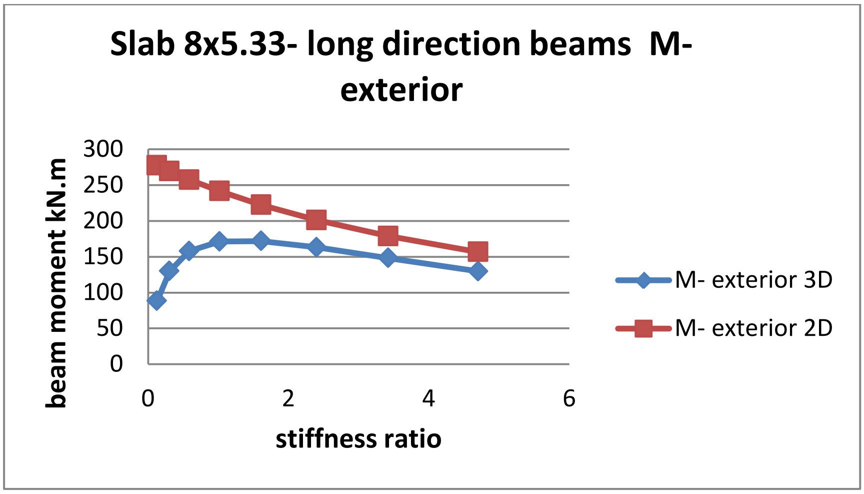
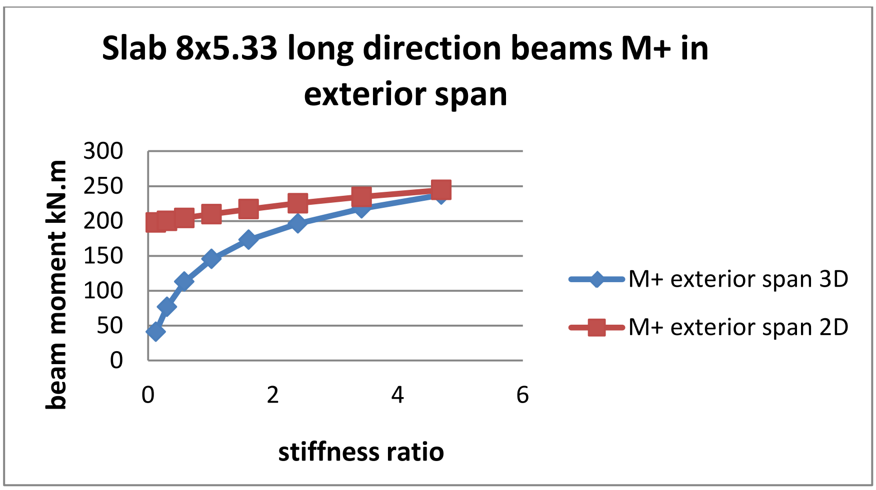


Figure 1 : relation between moments in beams determined by three and two dimensional analyses- E





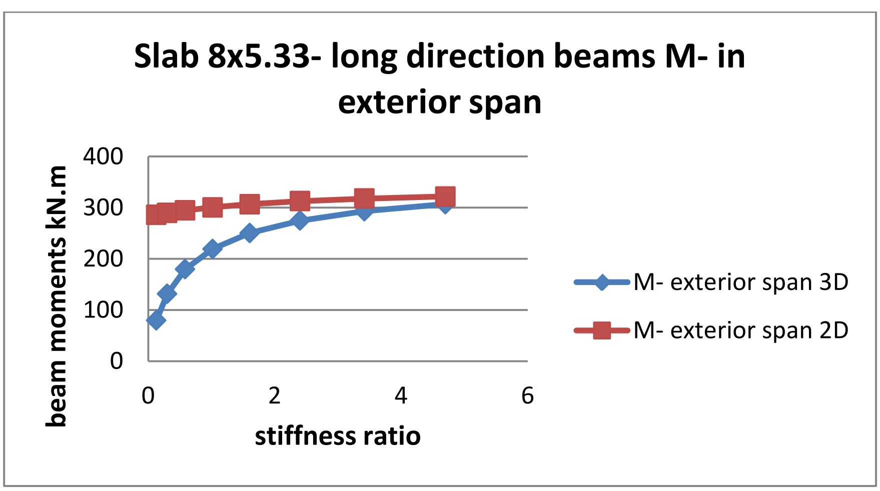


Figure 1 : Relation Between moments in beams determined by three and two dimensional analyses- F

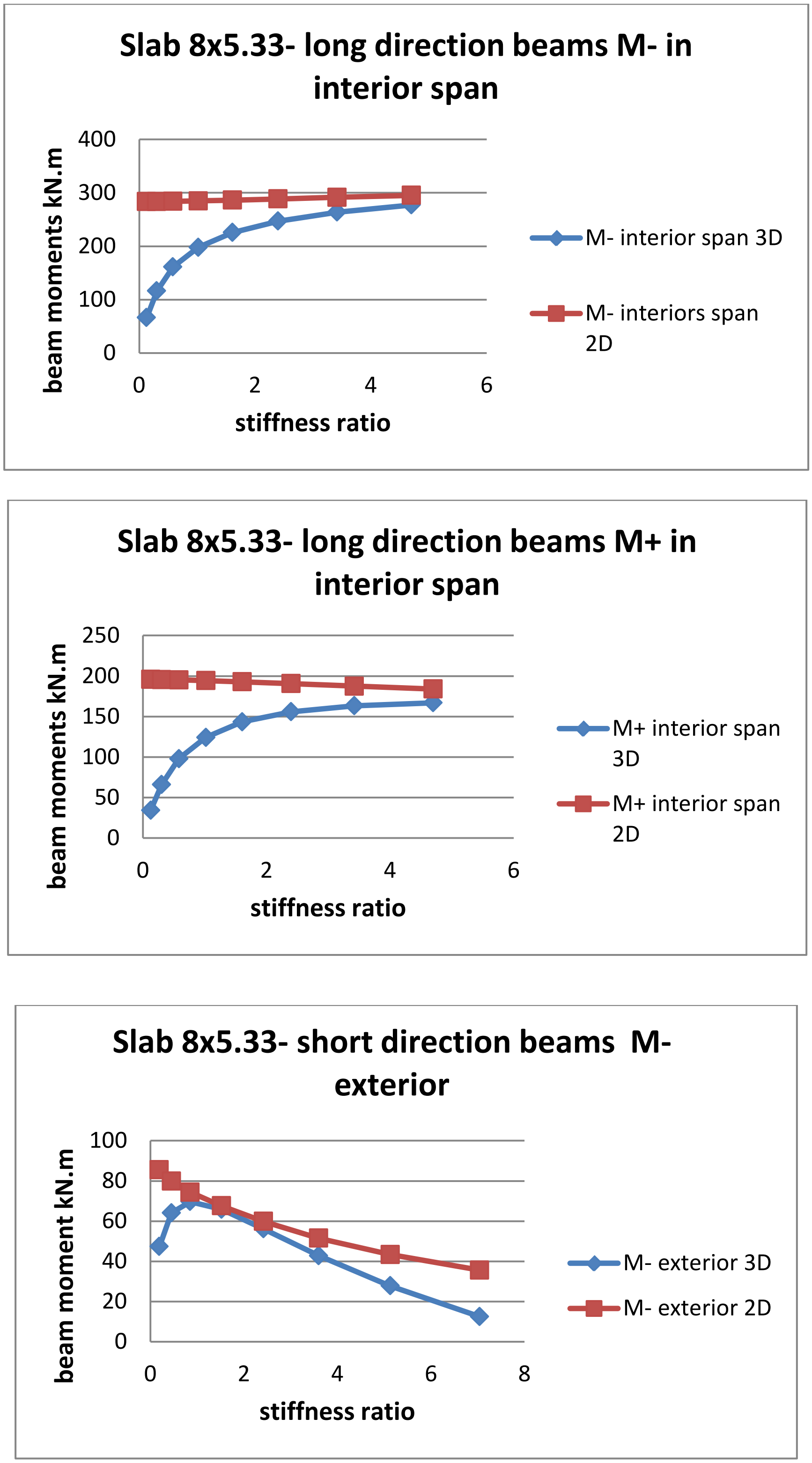


Figure 1 : Relation between moments in beams determined by three and two dimensional analyses- G

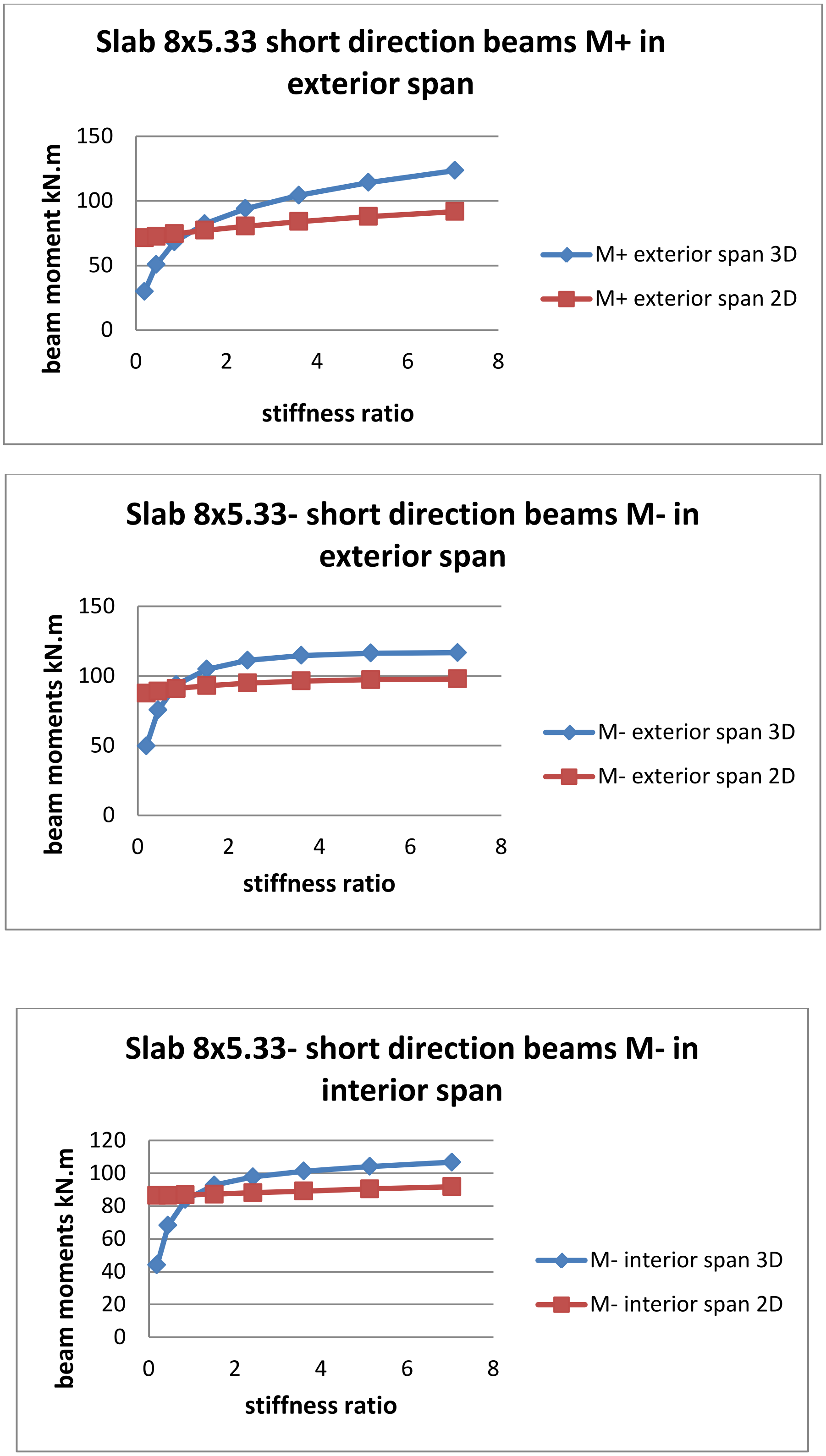


Figure 1 : Relation between moments in beams determined by three and two dimensional analyses- H

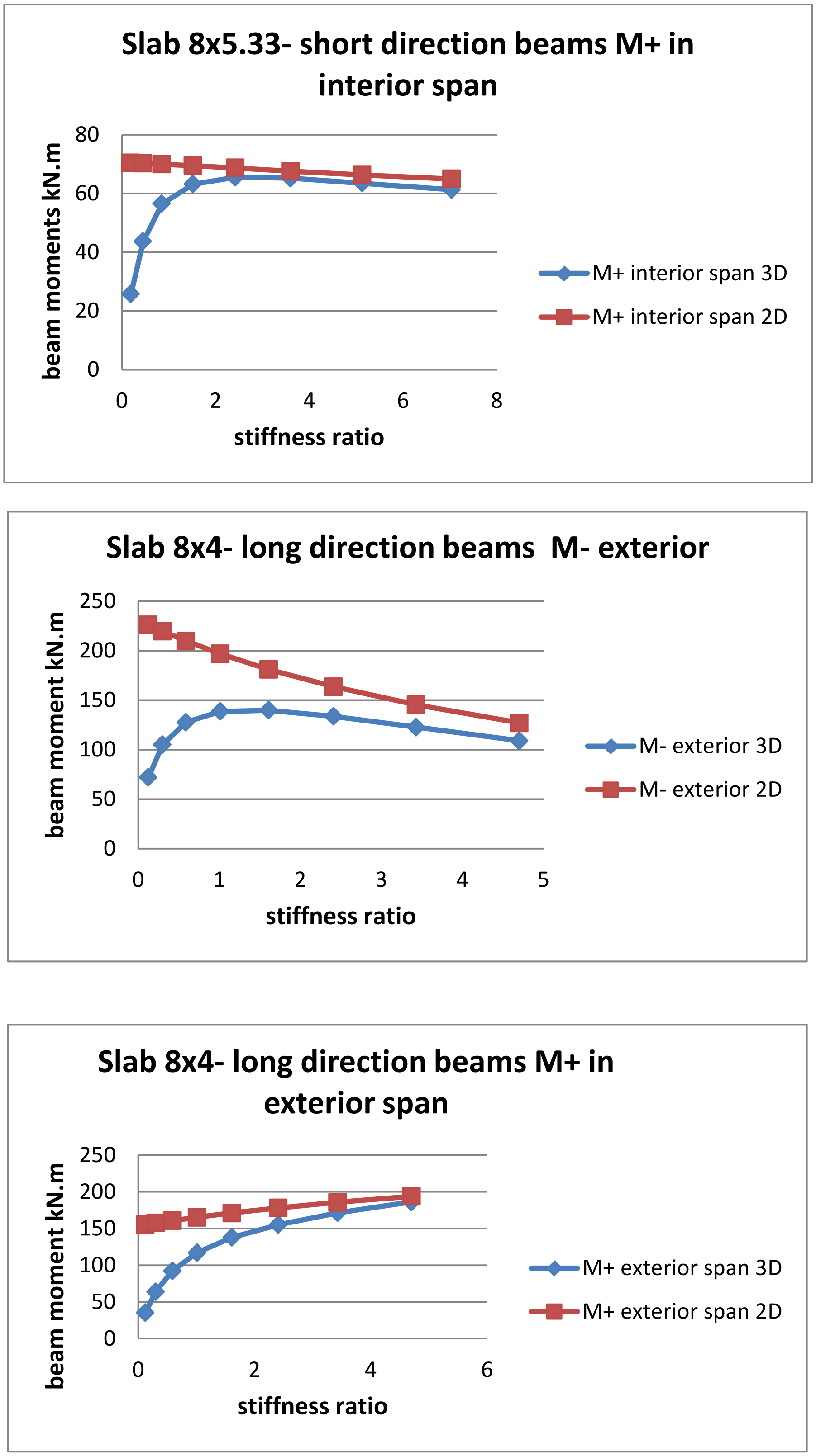
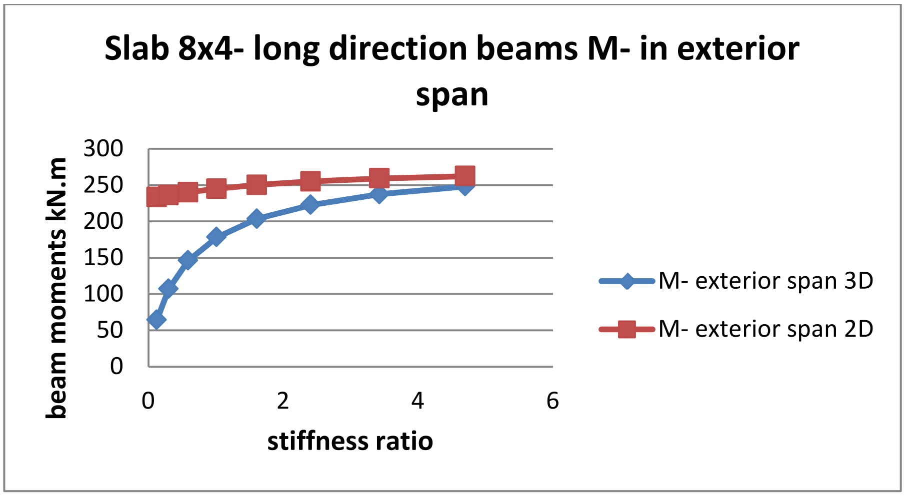
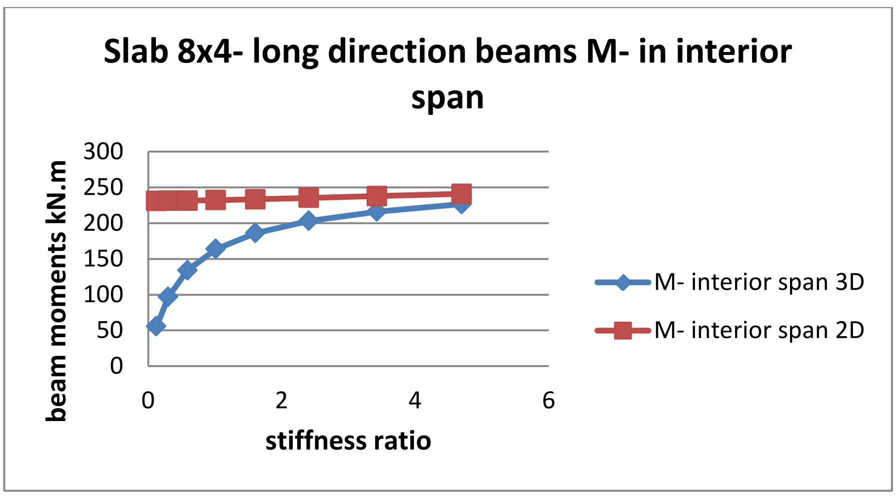


Figure 1 : Relation between moments in beams determined by three and two dimensional analyses- I





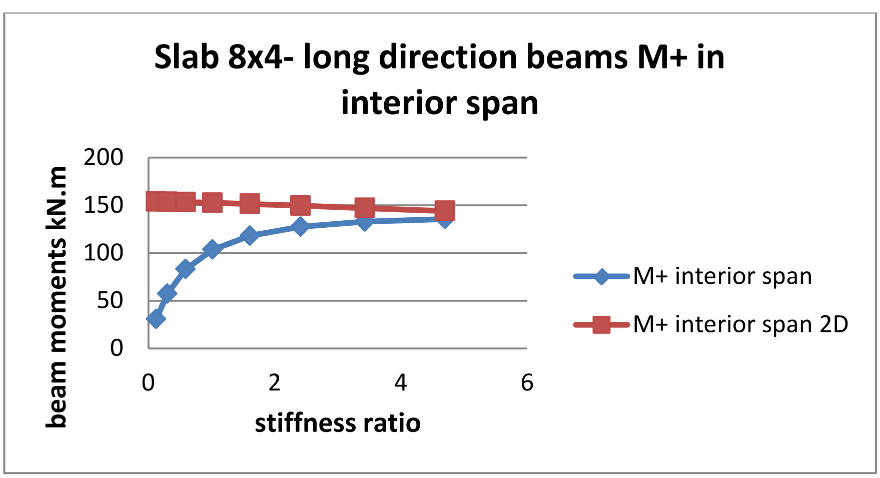
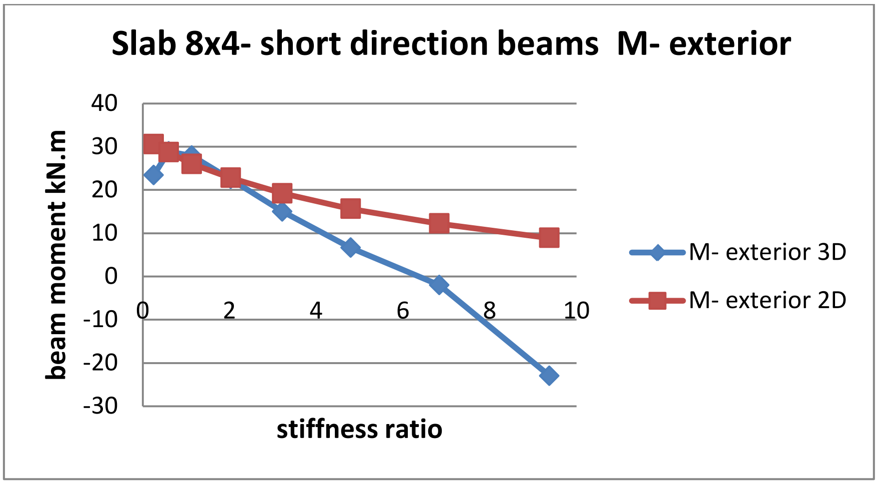
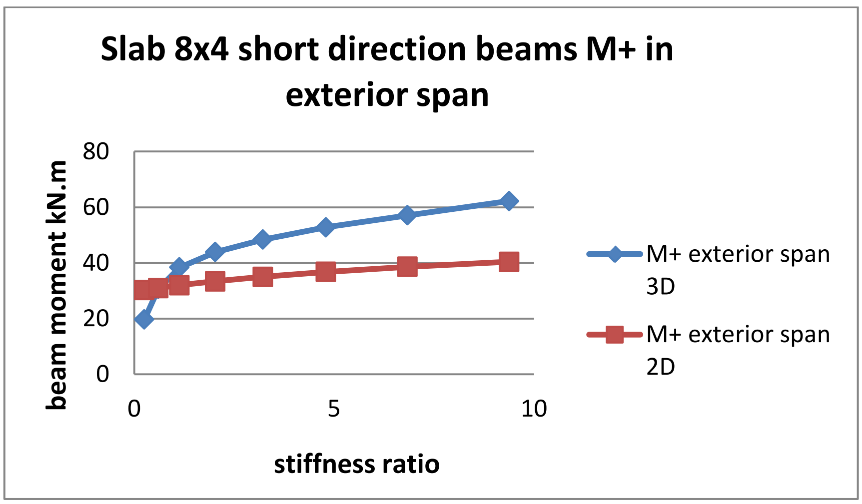


Figure 1 : Relation between moments in beams determined by three and two dimensional analyses- J





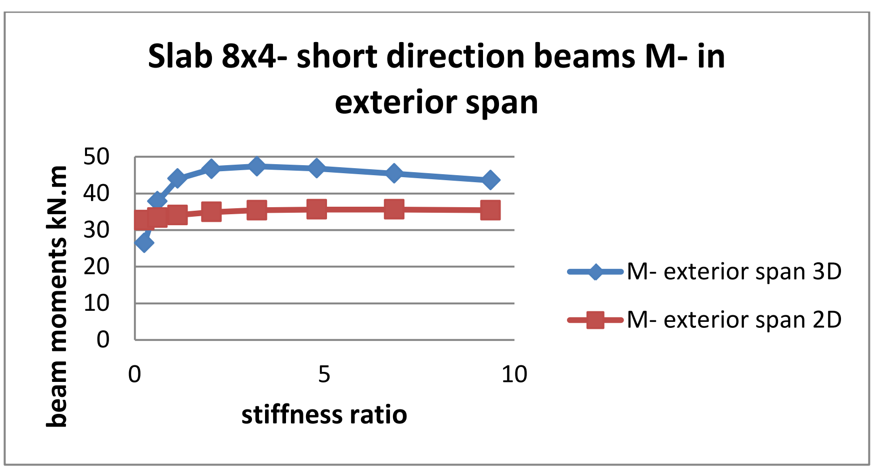
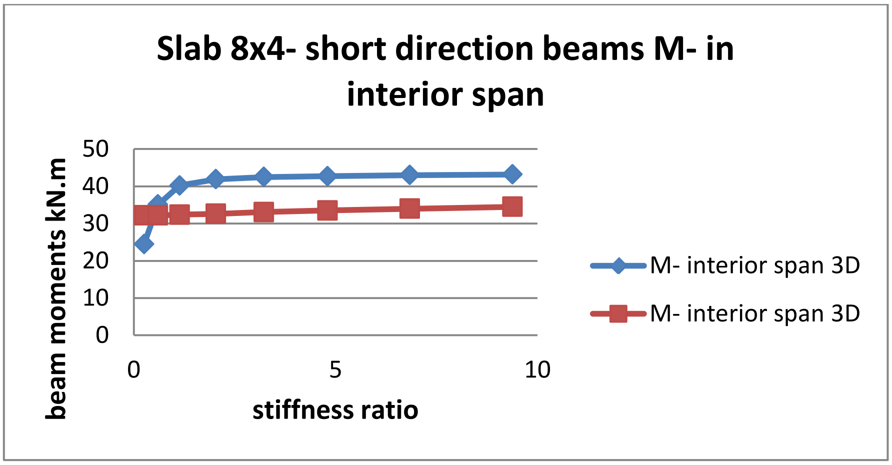


Figure 1 : Relation between moments in beams determined by three and two dimensional analyses- K



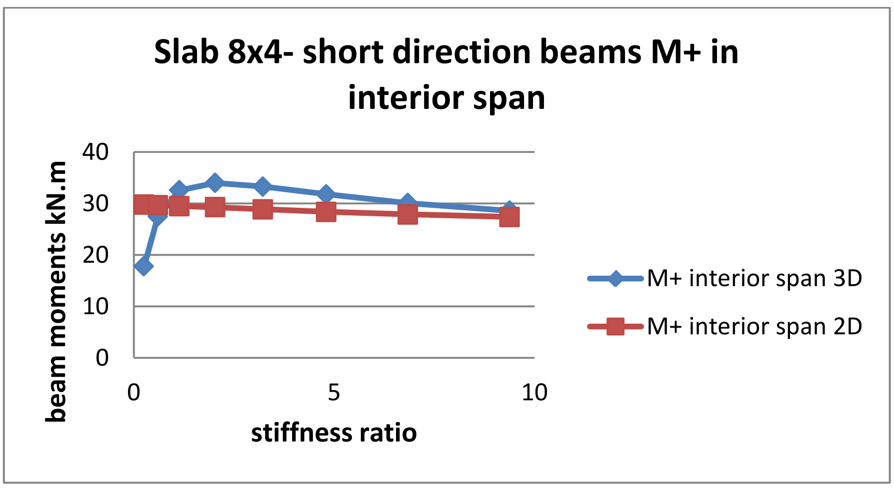


Figure 1 : Relation between moments in beams determined by three and two dimensional analyses- L

VII. Results and Recommendations

This paper shows that there is a difference in moments of beams in two way slabs between three dimensional structural analysis of slabs and the principle of 45 degrees-load-distribution. The results will be not far for a specific stiffness ratio. For rectangular panels, the difference between moment values will be acceptable for stiffness ratio not less than 3 for long spans, but in general the moments in beams determined by frame two dimensional analysis are slightly higher than the moments determined from three dimensional analysis. In short spans of rectangular panels, the difference between beam moments is acceptable at stiffness ratio of 2 for long span/short span of 1.25, 1 for long span/short span of 1.5 and 0.5 for long span/short span of 2. Also, the beam moments determined by three dimensional structural analysis will be larger than the beam moments determined by frame two dimensional structural analysis for stiffness ratios larger than these for short spans in rectangular panels.

In general, it is always recommended to use a finite element structural analysis program for analysis of two- way slab systems, as the distributions of internal forces in slab and beams depend on the span lengths and the relative stiffness between the structural members.

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