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In all fields, including here technical field, there will always be uncertainties [1,2]; from a quantity point of view uncertainties are a set of values that we can expect. To offer some examples in vehicle dynamic study there will always be uncertainties regarding weight value, rolling radius, rotational mass coefficient, drag coefficient, aerodynamic coefficient, front surface, transmission efficiency etc. [3]. Throughout the paper we call on operations with values intervals and on differential equations with coefficients that have values within certain intervals. Using the well known differential equation for straight movement, certain parameters are analyzed regarding their influences onto the vehicle dynamic behavior. The theoretical achieved results are than compared with results that are reached through real test runs carried onto a vehicle that has gasoline injection, onboard computer, transducers and other built-in actuators. The data offered by these sensors is collected using a data acquisition system.

Index terms— vehicle dynamics, uncertainty, interval analysis, interval differential equations.

n the technical literature it is being specified that uncertainties show the impossibility of making deterministic forecasting, thus using unique values [4,5]. This constitutes in fact the essence of uncertainty present analysis, which does not operate with unique values but with intervals of values. Likewise the technical literature there are several classification criteria for uncertainties; the current paper targets the study of vehicle dynamic behavior within the presence of parametric uncertainties.

In order to study the vehicle movement in the presence of uncertainties, first we start from the well known differential equation that describes vehicle longitudinal dynamic behavior [3]; considering movement on a horizontal road, speed being the unknown parameter v , the equation becomes:

$$m \frac{dv}{dt} = F - R - F_w \quad (1)$$

In relation (1) it is being noted: g -gravitational acceleration, η -rotational moving masses coefficient;
G a -vehicle gravity, M e -engine torque, i t -total transmission gear ratio, η t -transmission efficiency, r r -wheel rolling radii, f -rolling drag coefficient, k -aerodynamic coefficient, S -transversal vehicle surface.

Considering relations, first being true for a vehicle that is fitted with fixed shafts mechanical transmission:

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \eta_{\text{mech}}, \quad (2)$$

Where n represents engine speed, k s-shape coefficient, B -total vehicle width and H total vehicle height, equation (1) becomes:

In the differential equation (3) engine torque, its speed and vehicle speed are known from test runs for a certain experimentation. In this expression equation's coefficients are noted c_1 , c_2 and c_3 , defined by: $\frac{1}{2} \frac{3}{30} \frac{t}{s} \frac{a}{g} \frac{B}{H} \frac{g}{k} \frac{k}{f} \frac{G}{c} \frac{c}{G} \frac{c}{?} \frac{?}{?} \frac{?}{?} \frac{?}{?} = \frac{?}{?}$. (4)

These expressions present uncertainties over the next following parameters τ , γ , G , α , f , k and k_s (thus parametric uncertainties) because these aren't precisely known, but are better known within some intervals; some of these are adopted by the technical literature(τ , γ , f , k i k_s)

, and G a accordingly to the vehicle's technical specifications. As a result, every the 4th relation coefficients have values within certain limits, starting from a minimum value (m index) to a maximum one (p index), so expression (3) becomes: $1 \ 1 \ 2 \ 3 \ 2 \ 2 \ 3 \begin{bmatrix} ; \\ ; \\ ; \end{bmatrix} \begin{bmatrix} ; \\ ; \\ ; \end{bmatrix} \begin{bmatrix} ; \\ ; \\ ; \end{bmatrix} m \ p \ m \ e \ p \ m \ p \ c \ c \ c \ c \ c \ M \ n \ v \ v \ c \ v \ ? \ = \ ? \ ? \ , (5)$

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Where, accordingly to relation (4) $c c c g g B H g g g B H g f k k f k k G G c G c ? ? ? ? ? ? ? ? = = = = .$ (6)

So, expression 5 represents a differential equation that has coefficients situated between real limit values and thus in theory has a infinite number of solutions; other way put, the differential equation has its solution situated within a certain interval. This differential equation is being resolved with specific algorithms. The most widely spread calls onto Hukuhara [6] method of generalized differentiation.

Just the same as we can see from expression (5) and (6), the study of vehicle dynamics under uncertainty conditions calls on for operations with real intervals of values: addition, subtraction, multiplication and division. For example, for a random values of x and y :

$$\begin{bmatrix} ; \\ ; \end{bmatrix} \text{ x y x y x y x y x y x y x y x y?} = ? ? ? ? , (9)$$

And for division, on which relation (9) is used for parameters x and 1/y:1 , for 0 x x y y y = ? ? (10) b)

For start it is being considered that all 6 influencing parameters (t , θ , G , a , f , k and k_s) show certain uncertainties for their values, as figure ?? proves just that. For example, drag coefficient for rolling on dry asphalt varies between

$$[\cdot; \cdot] [0.012; 0.022],$$

m p f f f = so it varies with η = 83.3%, as we can see from the figure itself, where all the other 5 parameters are given. Figure ?? presents a test run S19 which was carried out with Skoda Octavia running of course on dry asphalt.

Proceeding in the same manner, figure ?? presents the variations at maximum and average speed for all 50 test-runs which were performed and from which we gathered data. Fig. ?? Figure ?? also presents the values for the differential equations coefficients (5), experimental curve of speed, the two curves afferent to the interval of achieved solutions (upper and lower or superior and inferior), maximum and minimum values for speed, as well as their variations (at average speed $V_m = 58.4\%$ and at maximum speed $V_{max} = 48.8\%$). Figure ?? also presents as an exemplification the values of speed at $t=26$ s: experimental value 54.5 km/h, upper value for that time is 60.9 km/h, lower limit 31.7 km/h. Figure ?? shows that throughout the 50 test-runs, the average speed varies up to 59.7% when the 6 parameters varies within the presented intervals from figure ??, and the maximum speed varies up to 47.2%.

The next example establishes the influence of the vehicle's weight on to its dynamic behavior, figure ?? for the same S19 (for comparison purpose), and figure ?? for all 50 test-runs; in tis case, the other 5 parameters were being equaled to their average values. As we can see from figure ??, the vehicle's weight varies up to 44.2% and figure ?? shows that overall the 50 test-runs the average speed varies up to 43.6%, with a maximum of 36.6%. Year 2014 Vehicle Dynamics Study under Uncertainty Fig. ?? Fig. ?? The final example establishes the influence of the aerodynamic coefficient k over to the vehicle's dynamic behavior: figure ?? for the same test-run S19 (for comparison purpose) and figure 6 from all experimental test-runs; in this case, the other 5 parameters were equaled to their average values.

As we can see from figure ??, aerodynamic coefficient varied up to 50%, when the average speed varied only with 42.8%, and the maximum speed up to 38.1%. As we can see from the graph and from the expressions described by (4), only c_3 varies within a certain interval, the other 2 remain constant. Adopting inverse dynamics principle, because only one parameter varies (aerodynamic parameter) it is possible to establish c_3 coefficient afferent to S19 test-run, meaning the value for the dynamic coefficient k_i . To this purpose, the diagram also presents the values for c_3 and k_i which represent the most closely curve to the experimental curve; this way $c_3 = 0.00026181$ and $k_i = 0.263$ were achieved for which the afferent curve only has an error of 0.38% compared to the experimental one. Similarly we proceed for any other influence parameter; as a result, figure ?? presents the influence of each of the 6 parameters (cases noted from 1 -6), as well as case 7 on which all parameters vary. Fig. ?? As we can see from the graph, there are parameters that have more significance through their variations as the variation of speed (f , G_a , k), so the effect is significantly less from a percentage point of view than the cause that triggered it. In exchange, we can see that frontal surface has a small variation (5.9%) comparative to speed (40.8% respectively 34.6%) so the effect is from the percentage point of view greater than the cause that triggered it; same conclusion may be applied to the transmissions efficiency as well as for the reduced mass coefficient.

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It may be concluded that the study of vehicle dynamic behavior in uncertainty conditions, as the real cases from normal exploitation occur, allows for the establishment of certain value intervals for those parameters that define vehicle dynamic behavior or their efficiency. This study calls on intervals of values for parameters, this being the



Figure 1:

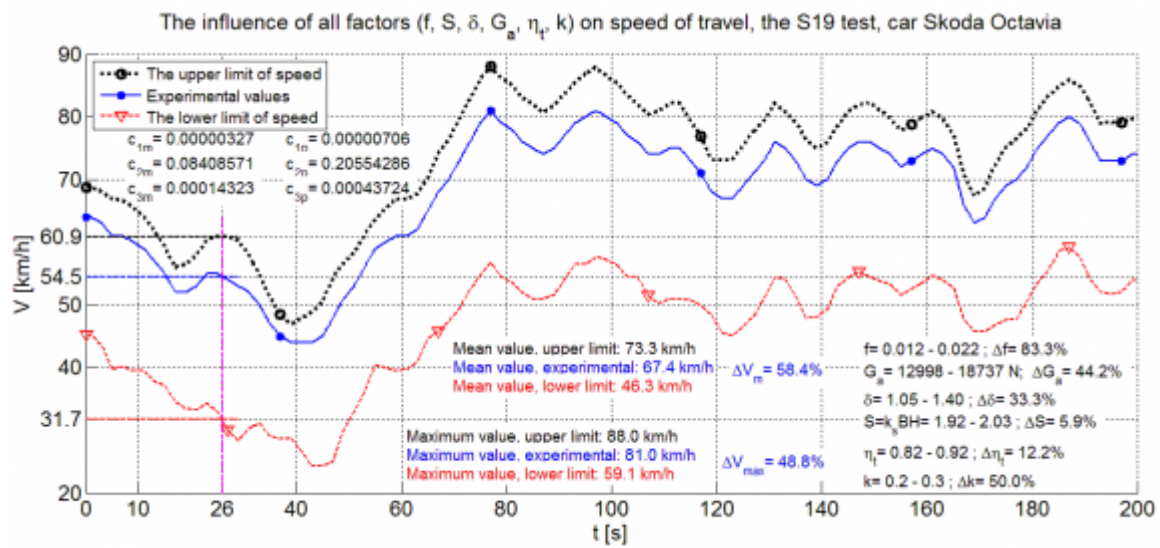


Figure 2:

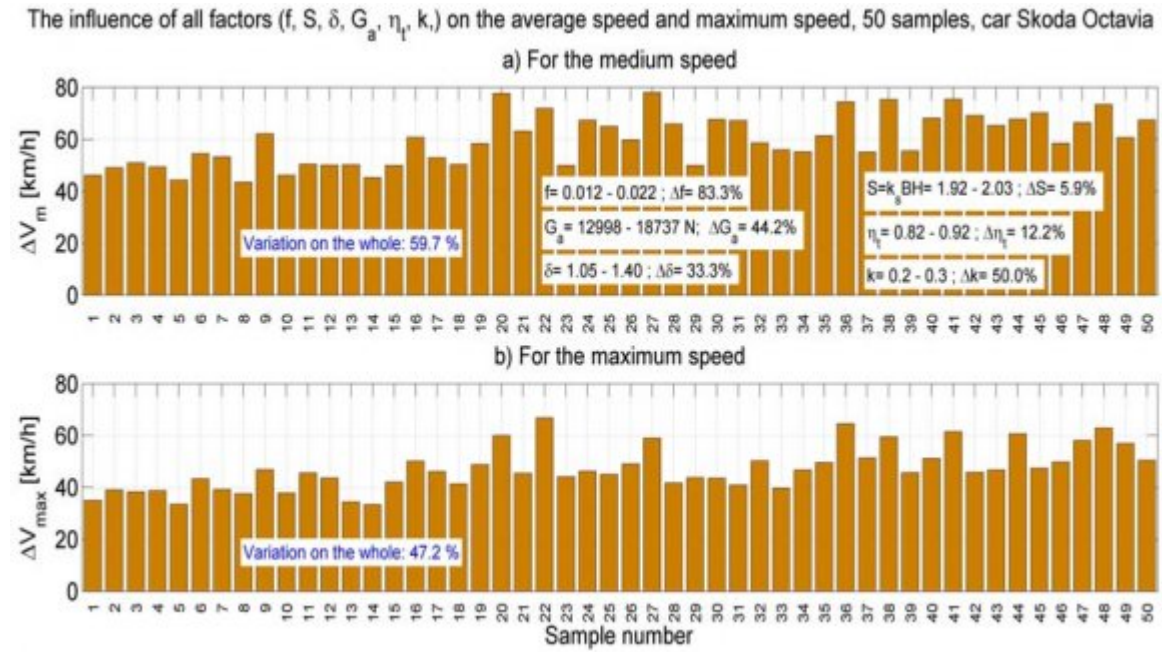


Figure 3:

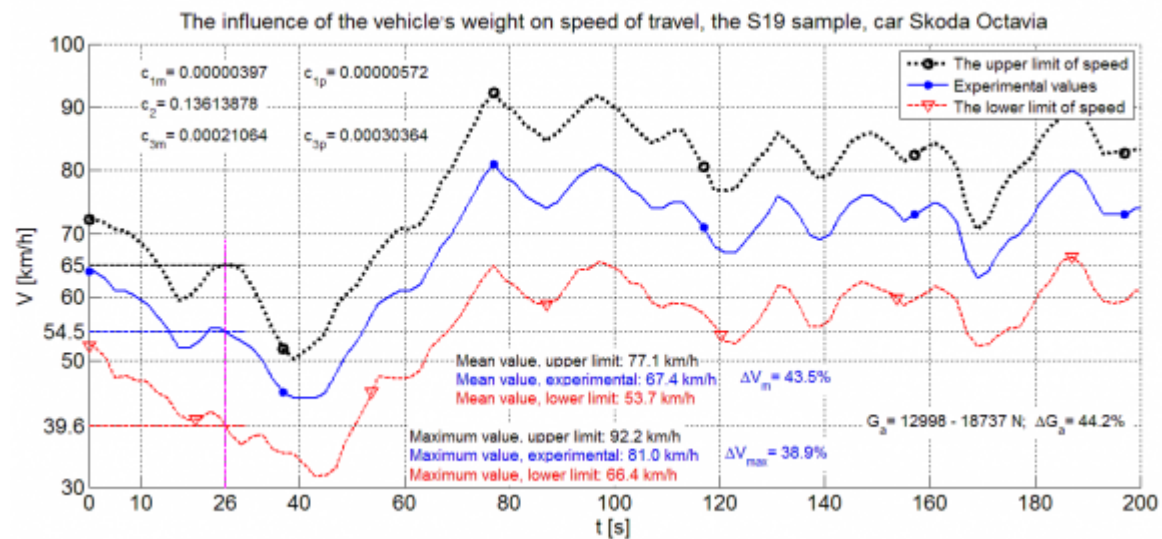


Figure 4:

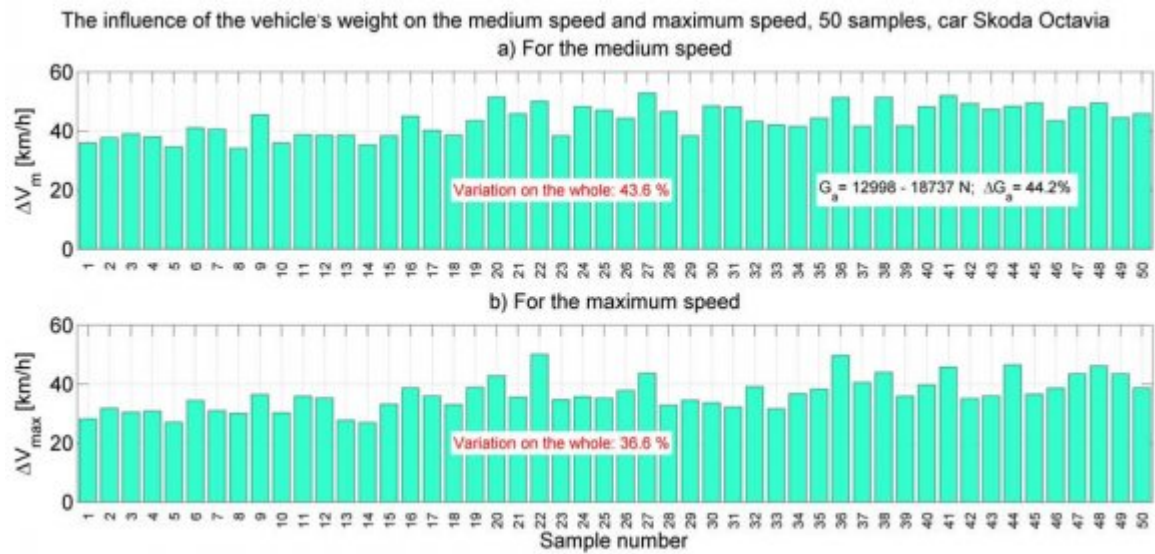


Figure 5:

1 m 30 tm p ap ; 1 p 30 tp m am ; 2 m m p ; 2 p p m ; 3 m m sm p

Figure 6:

103 main difference regarding the classic approach of vehicle dynamic behavior study from the specialty literature.
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