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I. Introduction

The existence in the literature of a large number of damage laws by fatigue for predicting lifetime of structures subjected to varying loads in service illustrates the difficulties of actually having a universal law to forecast lifetime [2]. Although the phenomenon of failure by fatigue of mechanical components subjected to cyclic mechanical strain was discovered more than a century by Wöhler, this type of damage remain still an outstanding event in terms of reliability. However, much progress has been made both in terms of its phenomenological characterization as of understanding of its mechanisms.

Among the proposed damage laws since several decades, the law of Chaboche has always seemed promising although its practical application raises some issues. The attractive aspects of this law are multiple: nonlinear evolution and accumulation of damage, influence of "small" cycles (cycles below the endurance limit) as soon as the damage is initiated, sequence effect (influence of the appearance order of cycles) and the effect of mean stress [3]. The implementation of Chaboche law faces an important challenge from the material parameters determination point of view. It requires the knowledge of the SN curve of the material which undergoes a linear regression in a reference particular and own to the law. Doing so, it significantly changes the SN curve and therefore deviates from the actual behavior of the material. This work is aimed toward the study and the resolution of this disadvantage. First are presented the issues of wedging of the Chaboche law and its impact on the estimated lifetime. An alternative is then proposed by modifying the formalism of the law in order to clear its sensitivity to the wedging field.

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II. Presentation of the Chaboche Law

The implementation of the law of Chaboche is presented here; it helps to highlight the issue faced during its wedging.

a) *Differential expression of the law*

The increment δD of damage by fatigue generated by δN amplitude cycles σ_a and of σ_m average value is given by:

$$\delta D = \left[1-(1-D)^{\beta+1}\right]^{\alpha}\left[\frac{\sigma_a}{M_0(1-b\sigma_m)(1-D)}\right]^{\beta}\delta N \qquad (1)$$

With: - b, β and M_0 are coefficients specific to the material; b is the slope of Haigh traction diagram, modeled linearly as following: $\sigma_A(\sigma_m) = \sigma_{-1}(1\text{-}b\sigma_m)$

- $\sigma_A(\sigma_m)$ is the amplitude of the fatigue limit of the material under average stress σ_m , σ_{-1} is fatigue limit of the material in alternating symmetrical tensile (R= –1).
- The coefficient α is defined by:
  - For a cycle located above the endurance limit of the material (and called "large" cycle), $\sigma_a > \sigma_A(\sigma_m)$:

$$\alpha = 1 - a\left\langle\frac{\sigma_a - \sigma_A(\sigma_m)}{Rm - \sigma_m - \sigma_a}\right\rangle = 1 - a\frac{\sigma_a - \sigma_A(\sigma_m)}{Rm - \sigma_m - \sigma_a}$$

  - For a "small" cycle, $\sigma_a \le \sigma_A(\sigma_m)$: a = 1

b) *Integration of differential law*

The cumulative damage is done cycle by cycle for all cycles encountered during loading. The lifetime of the material is determined by integration of the differential damage law [1] knowing that the initial damage is zero (virgin material) and reaches unity when the crack initiation appears. Distinct values of the parameter α depending on the type of cycle met ("small" or "large" cycle) generate two different integration formalisms of the damage.

i. *Integration of damage in case of small cycle*

The application of small cycle defined by (σ_a, σ_m), change the damage from Di value to Dj value. The integration of the damage in this case is given by:

$$\frac{1}{\beta+1}\ln\left(\frac{1-(1-D_j)^{\beta+1}}{1-(1-D_i)^{\beta+1}}\right)=\left[\frac{\sigma_a}{M_0(1-b\sigma_m)}\right]^{\beta} \quad (2)$$

By putting :

$$\begin{cases} X_i=\left[1-(1-D_i)^{\beta+1}\right]^{a} \\ X_j=\left[1-(1-D_j)^{\beta+1}\right]^{a} \end{cases} \quad (3)$$

The cumulative damage after applying the small cycle (σ_a, σ_m) is given by:

$$X_j=X_i\cdot e^{aM_0^{-\beta}(\beta+1)\left[\frac{\sigma_a}{(1-b\sigma_m)}\right]^{\beta}} \quad (4)$$

X appears as a functional damage whose characteristic is to cancel or be equal to unity at the same time as D. Note that if the level damage Di is zero initially, it remains after the application of small cycle. Small cycles do not contribute to material damage once the damage is initiated.

ii. *Integration of damage in case of large cycle*

The application of a large cycle defined by (σ_a, σ_m), moves the material damage from D_i value to Dj value. The integration of damage is then given by the following:

$$\left[1-(1-D_j)^{\beta+1}\right]^{1-\alpha}-\left[1-(1-D_i)^{\beta+1}\right]^{1-\alpha}=(1-\alpha_j)(1+\beta)\left[\frac{\sigma_a}{M_0(1-b\sigma_m)}\right]^{\beta} \quad (5)$$

Under constant amplitude, the integration leads to the SN curve expression within the meaning of the law of Chaboche; it is given by:

$$N_f=\frac{1}{aM_0^{-\beta}(1+\beta)K}\left[\frac{(1-b\sigma_m)}{\sigma_a}\right]^{\beta} \quad (6)$$

Where:

$$K=\frac{1-\alpha}{a}=\frac{\sigma_a-\sigma_A(\sigma_m)}{R_m-\sigma_m-\sigma_a}$$

N_f is the number of cycles at crack initiation within the material under constant amplitude loading defined by the (σ_a, σ_m).

By repeating,

$$X_i=\left[1-(1-D_i)^{\beta+1}\right]^{a} \quad \text{et} \quad X_j=\left[1-(1-D_j)^{\beta+1}\right]^{a},$$

the equation (5) of the integration of damage becomes:

$$X_j^{K_j}=X_i^{K_j}+\frac{1}{N_{fj}} \quad (7)$$

III. Wedging Principle of the Law

The damage cumulative observed in the case of a small cycle and a large cycle (equations (4), (6) and (7) respectively), requires the determination of the material parameters β and $aM_0^{-\beta}$. This step is the wedging of the law.

The application of an alternating symmetrical tensile-compression stress with constant amplitude $\sigma_m = 0$, $R = -1$) simplifies the expression [6] of the material lifetime N_f:

$$N_f=\frac{1}{aM_0^{-\beta}(\beta+1)K}\left(\frac{1}{\sigma_a}\right)^{\beta} \quad (8)$$

Giving:

$$\ln(N_f.K) = -\beta\ln\sigma_a - \ln(aM_0^{-\beta}(\beta+1)) \quad (9)$$

or again:

Y = AX + B

With: $\begin{cases} Y=\ln(K.N_f) \\ X=\ln\sigma_a \\ B=-\ln\left(aM_0^{-\beta}(\beta+1)\right) \\ A=-\beta \end{cases}$

The values of σ_a and N_f to be used for the determination of these parameters are those points of the SN curve of Bastenaire in symmetrical alternating traction. The selection of all known points of the SN curve or part of them only defines what is called the wedging windows of the law.

The SN curve subtended by the damage law of Chaboche is then based on equation [9] a straight line in reference (ln σ_a; ln(K.Nf)), reference that is later called the wedging space. To obtain both $aM_0^{-\beta}$ and β coefficients used in the damage integration, the points of the SN curve of the material (expressed using Bastenaire model) are placed in the wedging space (Figure 1). A linear regression of all those points or of part of them leads in practice to the determination of $aM_0^{-\beta}$ and β.

The main problem of wedging of the Chaboche law lays in the fact that the Bastenaire SN curve of the material is not linear in the wedging space, even if we consider as window all the curve or only part of it. No specific guidance is made by the author of the law [1]. This will practically generate under uniaxial fatigue with constant amplitude a lifetime prediction significantly different from those projected by the SN curve intrinsic of material. The use under uniaxial fatigue with variable amplitude, of a law that does not match the SN curve of material in fatigue under constant amplitude is a failure in its principle.

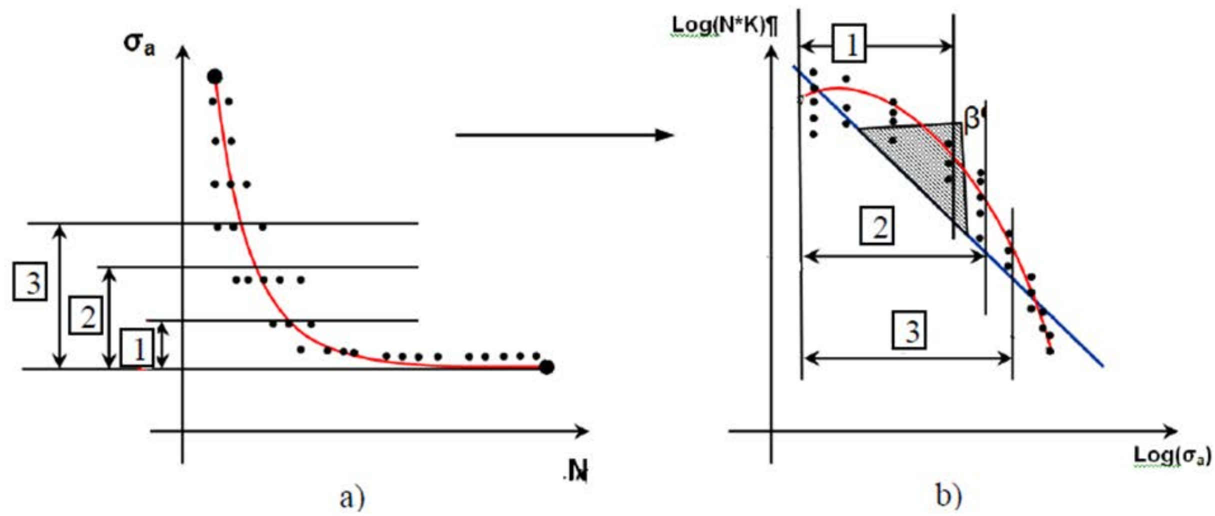

Figure 1 : Wedging principle of the law of Chaboche [5] : All points of the SN curve of Bastenaire are placed in the wedging space. Wedging by linear regression can be applied to all points or only points on a fraction of them (wedging window)

IV. Proposition of New Damage Law

Chaboche law was mainly applied, often successfully, to sequences containing two loading blocks commonly known as sequences High-Low or Low- High depending on the relative intensity of successive levels of stress [1].

The disadvantage of the law vis-à-vis its wedging makes the law not useful on its state. Depending on the wedging window chosen, the calculated lifetimes vary very importantly. The failure of the formalism of Chaboche law is in reality due to the fact that the SN curve of the material within the meaning of the law is a straight line in the wedging reference, which does not correspond to the usual SN curves of materials (Bastenaire model used here, but also those of Wöhler, Basquin or Stromeyer [4].

In other words, the problem of the sensitivity of the law vis-à-vis the wedging window used is the fact that the actual SN curve of the material does not match the one stipulated by the law, namely a straight line in the wedging space. Based on this observation, the main goal of the work done is to modify the formalism of the law to find the actual SN curve of the material when is realized the cumulative damage under constant amplitude loading.

The proposed model uses here also stress as mechanical state parameter, and it ties the increase δD of damage D to the number δN of cycles that generated it under the following differential form:

$$\delta D = \left[1-(1-D)^{\beta+1}\right]^{\alpha} x \quad \frac{1}{(1-D)^{\beta}}\left[\frac{R_m-\sigma_a-\sigma_m}{\sigma_a-\sigma_A(\sigma_m)}\cdot\frac{\sigma_{oo}(\sigma_m)-\sigma_D}{e^{-\left(\frac{\sigma_{oo}(\sigma_m)-\sigma_D}{B}\right)^{c}}}\right]\delta N \qquad (10)$$

The variables used in this law are similar to those used in Chaboche model. The only new variable, denoted σ_∞, represents the Y axis at origin of Goodman diagram passing through the point representative of the cycle analyzed. This variable is given by the expression: $\sigma_\infty = \sigma_a - b\sigma_{-1}\,\sigma_m$; the slope b of Goodman line is usually determined from the fatigue limits in symmetrical alternated traction and in repeated traction (σ_{-1} and σ_0 respectively). The integration results in the SN curve modeled by Bastenaire expression :

$$N_{fj} = \frac{1}{a\,(\beta+1)(\sigma_{oo}(\sigma_m)-\sigma\ )}\cdot e^{-\left(\frac{\sigma_{oo}(\sigma_m)-\sigma_D}{B}\right)^{c}} \qquad (11)$$

B and C are the Bastenaire model parameters calculated by smoothing experimental points used in the determination of the SN curve of the material.

V. Comparison of the New Proposed Law with the Chaboche Law Wedged on Several Different Windows and the Milner Law

Life Forecasts are made for steel 20MV6 subject to a sequence of variable amplitude loading CARLOS LATERAL used as reference in the automotive industry.

The wedging of the new law on the Bastenaire SN curve of the material lead to outstanding forecasts, especially for significant levels of stress, to those obtained by the law of Chaboche. Forecasts obtained from Miner linear law is also shown (Figure 2). The proposed new model provides intermediate life forecasts between those of Miner law and those of Chaboche law in the field of limited endurance.

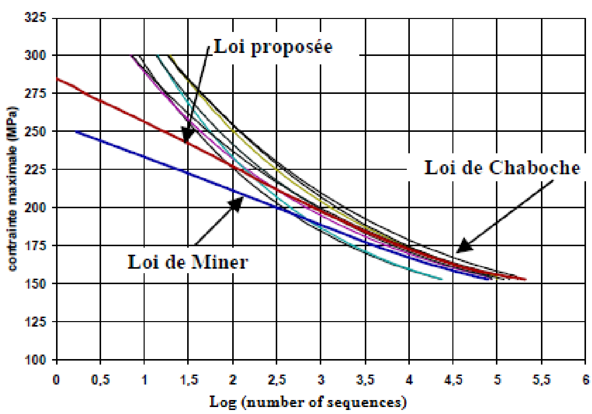

Figure 2 : Gassner curves obtained by using Chaboche law, Miner law and the new proposed damage law for the material 20MV6 subjected to CARLOS SIDE loading sequence [4]

VI. Conclusion

A new formalism of non-linear damage law by fatigue has been proposed. It is an alternative to the law of Chaboche because it eliminates the problem of wedging, that Chaboche law faces, on the material data that makes the S-N curve. The new law gives lifetime forecasts coincident with the SN curve of material in uniaxial fatigue under constant amplitude; it keeps the essential characteristics of the Chaboche law: sequence effect, the influence of the mean stress and non-linear accumulation of damage.

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