

1 Histogram Filter with Adjustment of the Smoothing Parameter
2 based on the Minimization of the Chi-Square Test

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6 Abstract

7 For the formation of adequate models of objects of statistical research, with the possible high
8 cost of a measuring experiment or the process of obtaining data, fast and correct
9 identification (recognition) of the probability distribution density (PDD) based on the
10 construction of simple histogram estimates is required. The requirement for rapid
11 identification can be considered equivalent to having a limited and small amount of data. The
12 article proposes a theoretically substantiated method for constructing a histogram filter (HF),
13 which is a linear combination of the amount of data in adjacent intervals with constant weight
14 coefficients, which can be expressed in terms of a single coefficient k - the smoothing
15 parameter. The estimation of the smoothing coefficient is based on the minimization of the
16 modified chi-square test. The theorem given in the article establishes that the value of the
17 mathematical expectation of the chi-square test, after applying the HF, decreases by k times
18 compared to the standard mathematical expectation of the criterion with a unit inclusion
19 function. The smoothing coefficient is determined by a complex dependence of the number of
20 data, parameters of the identified PDD (Fisher information coefficients of the first and second
21 order) and HF (number and width of grouping intervals). The article shows that the
22 relationship between the number of data, the number and width of grouping intervals is
23 non-linear and has only a numerical solution. The considered examples of modeling the work
24 of the HF characterize the effectiveness of the identification of the PDD, the expediency of its
25 application in scientific and applied statistical research.

27 **Index terms**— distribution density, density identification, histogram filter, smoothing factor.
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29 ?????????? ?????? ?????????? Д????, ?????????? ?? ?????????? ?????????? ?????, ??????????
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32 ? ?????? ?????????????? ?????, ?????????????? ? [6]. ?????? ?????, ?????????????? ?????????? ?? ?????????
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48 ?????????????? ??????????? ?????????????, ?????????, ????????? ??????????? ??????????? ?????????????? ??????????
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52 ???? ??????? ????????? ? ??????????? ?????????????????? ????????? ? ??????????? ??????????? ?????????????? ?? ??????
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56 ????? ??????? ?????????? ?????????? ?????? { j i x . 1, i n = ? ?????????? ?????????? ?????????? ??????? ? m
57 ?????????????????????? ? ?????????????? ???? ? ?????? ?????????????? j A , 1, j m = ?????? ?????? 1 / x j j X X R m + ? =
58 ? = . 1 max max m i i X x x + = = . 1 min min i i X x x = = . ??? j X -????????? ??????????????, max min x R x x
59 m = ?
60 =????????? ??????????? ??????. ?????????? ?????????? ??????????????? ?????????, ??????????????? ??? ?????????????? ?????????
61 ?????????????? ?????????????, ????????? ?????????????? ?????????? () j i x μ , 0 1 j ? μ ? , ? ?????????? ?????????????? 3 x μ ?
62 = ? ? ?????????? ?????? ?????????????????? ?????????????????? ??????? ??????????? ?????????????????? j A .
63 ??? ???? ?????? ?????????? ?????????? ?????????? ?????????????????? ? ??????? ?????????? ??????????????: () (1 , 1 , 3) / (1) (1)
64 1 , 2 , -1 , 1 , , 1 , . t m m j j t t j j t m t m j t m j t m ? ? ? + ? + = ? ? μ = = ? ? μ = = ? ? ? ? (1)
65 ?????????????????? ?????????? , j t μ ??????????????, ?? ??????????? ?? ?????????? ?????????? ??????????: ??? ?????????
66 k????????????? ??????????????. ?????????? ????????????? (1) ??? ???? ?????????????? ??????????????????. ?????? ???????????,
67 ?????????????? ?????????? D"?" ?????? ??? () (3) / (1) (1 1 1) , 2 , 1 , (1) , 1 , , (1)
68 / 2 . j j j j j j j m m m m u v k v j m u v v j m k ? + ? ? + ? ? ? = ? + + ? = ? ? = ? ? + ? = ? ? ? = ?
69 ? (3)
70 ?????? ???????, ??????? ?????????????? ?????????????? D"?" ?????????? ? ?????????????? ??????????????? ?????????????? ??
71 ?????????????? ? ?????? ?????? ? ?????????? ??????????? ?? ?????????????????????? ???.. ?????????? ? ?????????? ??????????
72 ??????? ??????????????? ?????????????? ?????????? ??-????????? ? ?????????? ??????j v ? ?????????? 2 () v ? ?? ?????? 1 1 j j
73 j j u v k v ? + = ? + + ? ??? 2 , 1 j m = ? ? () (3) / (1) (1)
74 (1) j m m j j m m u v v ? ? ? + ? = ? ? + ? ??? 1 , j m = , ?????????? 2 2 ?? 1 () / min m j j j k j u n p n p
75 = ? ? ? = ? ? ? ? (4)
76 ?????????? ?????????????????? ????????? (4) ?????????? ? ??????????? ??? ??????????????? ??????????????? ?? ?????????? ?????????1
77 2 ??? 1 1 1 2 1 1 1 2 / () / 1 2 / / , m m j j j j j j j m m j j j j j j j k U n v n p U n U p p p p n v U n ? = = ?
78 = = ? ? = + ? = ? ? ? ? ? ? ? = + ? ? ? ? ? ? ? ? (5)
79 ??? 1 1 2 j j j j U v v v ? + = ? + -????????? ?????????? ?????????? ?????????? ??? ?????????? 2 , 1 , j m = ? ? () (3) / (1) (1) j j j m m m m U v v ? ? ? + ? = ? + ??? ?????????? 1 , j m = ; 1 0 m j j U = = ? ; j p -?????????????????
80 ??????????????????
81 ??????????????????
82 ?????????????????? ?????????????????? ?????????? (5) ?????????? ? ?????????????? 0 1 4 1 2 2 2 4 1 1 1 5 , 0 5 (0 , 2 5) 0 1 , 5 () 2 x x x k n I I I m ? ? ? ? + ? = ? + + , (6) ??? ()
83) 2 * * * 1 / R I f f f d x ? = ? , () 2 * * * 2 / R I f f f d x ? ? = ? -????????????????????????????????? ???, ?????????????????
84 ?????????????? ??????? ?????????? ?????????? ?????????? [7] , 0 lim / x j x m f p ? ? ? ? ? = ? ? ? -????????????????? ???,
85 * , 0 lim / x j x m f v n ? ? ? ? ? = ? ? ? -????????????? ?????????????????????? ???, R f d x = ? ? -?????????????????
86 ??????????????????
87 ?????????? ?????????????? ?????????????? ?????????? (6) , ??? ???? ?????????????????? ?????????????? ?????????????,
88 ?????????? ? (5) . ?????????????? ???????????????-????????????????????? ??????, ?????????????? ?????????? ? ??????????????
89 ?????????????????? ?????????? ? ?????????? ?????????????? : 1 0 * lim / () / (/) x m j j j m R U n f R d p m f x ? ? ? =
90 ?? = ? ? , 1 1 1 1 1 2 1 () 2 (1) 2 (1) m m j j j j j j j m m j j v U v v p p p v U v U n p m m n n n ? ?
91 + = = ? ? ? ? + = ? + ? + + = ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? M M (7) * * * * * 1 1 2 2 * * * * * 1
92 1 1 1 2 2 j j x x x j j f f f f f f a b f f f f f f ? + ? ? ? ? ? ? ? ? ? ? ? ? ? ? = ? ? + ? = + ? + ? ? ? ?
93 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? , 2 2 2 2 1 1 1 1 2 2 2 1 1 1 1 1 1 2 1 2 2 2 0 1 0 0 4 lim lim 2 2 () () 2 lim
94 lim 4 () (1)
95 x x x m m j j j j m j j j j j j m m m j j j j m U v v n n v p v v v v v v n n n M p p p a b n m
96 p ? ? + ?? ?? = = ? ? ? ? ? ? ? ? ? ? + ? ?? ?? = ? ? ? ? + = ? ? ? ? ? ? ? ? ? ? ? ? ? ? + ?
97 ? ? + ? + ? + + ? ? ? ? ? ? ? ? ? ? ? ? ? ? + + + ? ? ? ? ? ? ? ? M M M M [] 2 * 4 * 4 * 1 2 2 2 () 8 6 (1)
98 2 (1) 0 , 5 (1)
99 x x x M a b n m I m I m I n + ? = = ? + ? ? + ? ? + ? 1 / 3 1 k ? ? .
100 ??-??????, ?????????? ?????????? ?????????????? ?????????????? (5) ? ??????? (4) ??? ?????????? ?????????? ??????????
101 ?????????????????????? 1 2 2 D"?" 2 2 1 1 / () / () m m j j j j j j u U n v U p p n v ? = = ? ? ? ? ? ? ? ? ? ? =
102 ? ? ? ? ? ? ?
103 , ?? ?????????? ?????????? ?????????????? ?????? ?????????????????? ?????????????? ?????????? ??-?????????
104 () () ?????????? (9) ??????????, ?????????? ?????????? ?????????? ?????????? ? ?????? ??????. ?????? ?????????? (9)

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106 ?????????????? ?????????????? ?????????? ?????????????? ??? ? Ð??. ???, ? ?????????, ?????????
107 ??????? ?????????? ?????, ??????????? ?????????? ?????????????? ??? ?????????? ? ?????????????? ?????????? ?
108 ?????? ? ??? ??? ??? ????????? ????? ??????? ??????? ??????? ?????????? ??????? [9,10], ?????????? ???????????
109 ?????? ?????? ? ??????? ??????? ?????????????? ?????????????????? ???2 2 ?? k ? = ? ? ? , ( )2I ? 1 2 1 ( ) 1
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111 2 2 e ( )2 ( )D Erf Erf Erf Erf D Erf D D D ? ? ? ? ? ? ? ? ? ? ? ? + ? ? ? ? ? 2. ?????????????? ??????????:
112 / 2 x e ?? ? 1 I ? 2 1 I ? ( ) ( ) 2 1 1 ? ? ? + ? ? ? 2 I ? 4 2 I ? ( ) ( )4
113 ? ?????????? ??????? ???????, ? ?????? ?????????? ??????? ?????????????? ?????????? ?????????? ??????????
114 ??? ? Ð??. ?????????? (9) ?????????? ??????????. I I I n K m ? ? ? + ? ? = + , ??? / x R m ? = , 0 0 1 0 ?? ??
115 ( ? ) ( ? 1 3 ) K ? ? ? = . 0 1 ?? 0 ? k ? = (9)
116 ??-?????, ??? ?????????? ?????????????? ?????? ?????????????????? ?????????????????? ( ) 0 a n x b x x K + + ? = ,
117 0,25 b c = , 1 1 ( ) 2 ( ) a n R I n b ? = .(10)
118 ?????????? ( 10) ?????????? ? ?????????? ?????????? ?????????? ?????????????? ?????????? ??????????
119 ?????????????????? ??? ? Ð??.
120 ??-?????, ?????????? ?????? ?????????? ?????????????? (6) ? ?????????? ?????????? ?????????, ??? ?
121 ?????????? ?????? ?????? ?????????? ??????? ??????????, ?????????? ????????? n. ?????? ?????????? (9) ?????
122 ?????????????? ? ??? ( ) 0 2 4 4 2 1 I m n K m R ? =(11)
123 Ð??.??? ????? 1 m » ??????? ?????????????? ?????????????? ??????? ?????????????? ?????????? (11): 4 4 0 2 0,5
124 1 / n I m R K m ? ? .
125 ?????????? ?????????????? ?????????? ?????????? ?????????????????? ?????????? Ð??.? ? ?????? ??????????????
126 ? ?????? (?? ????? ??????? ??????) ?????????????? ?????????????? ??????? ?????????????? ? ??????????????
127 ??????????, ?? ?????????? ?????????? ??????????. ?????????? ?????????? Ð??.? ?????? ?????????????? ?
128 ?????????????? ?????????? ?????????? ??????????, ?????????, ? ??????? hist, histfit ?????????? Matlab.
129 ?????????? ?????????? ?????????????? (?????????????) ??? ?????????? 1. ?????????? ??????? ??????, ??????? n,
130 ?????????????? ??????? ??????? R. ?????????????? ?????????????? ?????? ?????????????????? ?????????? ??????????
131 ??-????? ?? ?????????? ??????? ?????????? ?????????????? ??????? ? ? ?????????????????? Ð??.? ?????
132 ?????????????? ?????????????? ?????????????? ?????????????? (5), (6). ?????????? ?????????? ?????????????? ?????????????
133 ??????? ? ? ??????????: ?????? ??????, ?????? ?????????? ??????????????, ?????????????????? ??????????????
134 ?? (?????? 1). ?????????? ?????????????? ?????????????? ?? ?????????? ?????????? ?????????? ??????????
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$$\begin{cases} \mu_j(x) = \left\{ k \text{ для } A_j; \alpha = (1 - k) / 2 \text{ для } A_{j-1} \text{ и } A_{j+1} \right\}, & j = \overline{2, m-1}, \\ \mu_j(x) = \left\{ (1 - \alpha) \text{ для } A_j; \alpha \text{ для } A_{(j-m)(m-3)/(m-1)+(m-1)} \right\}, & j = 1, m, \end{cases}$$

Figure 1: . 1 I ? * 2 I ???????? 1 :

I	№3. Логистическая плотность: $\alpha \operatorname{sech}^2(\alpha x) / 2$
I_1	$4\alpha^2/3$
$I_{1\gamma}$	$\frac{4}{3} \alpha^2 \tanh \left[\frac{2\sqrt{3}\alpha \operatorname{ArcTanh}[\gamma]}{\pi} \right]^3$
I_2	$16\alpha^4/5$
$I_{2\gamma}$	$\frac{1}{5} \alpha^4 \operatorname{Sech} \left[\frac{2\sqrt{3}\alpha \operatorname{ArcTanh}[\gamma]}{\pi} \right]^5 \times$ $\left(30 \sinh \left[\frac{2\sqrt{3}\alpha \operatorname{ArcTanh}[\gamma]}{\pi} \right] - 5 \sinh \left[\frac{6\sqrt{3}\alpha \operatorname{ArcTanh}[\gamma]}{\pi} \right] + \sinh \left[\frac{10\sqrt{3}\alpha \operatorname{ArcTanh}[\gamma]}{\pi} \right] \right)$
	№4. Коши плотность: $s(s^2 + x^2)^{-1} / \pi$
I_1	$0.5s^{-2}$
$I_{1\gamma}$	$\frac{4 \arctan \left[\tan \left[\frac{\pi\gamma}{2} \right] \right] - \sin [2\pi\gamma]}{4\pi s^2}$
I_2	s^{-4}
$I_{2\gamma}$	$\frac{48 \arctan \left[\tan \left[\frac{\pi\gamma}{2} \right] \right] + 24 \sin [\pi\gamma] + 6 \sin [2\pi\gamma] + 8 \sin [3\pi\gamma] + 3 \sin [4\pi\gamma]}{24\pi s^4}$

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Figure 2: ????????????? ????????? 1 .© 1 I

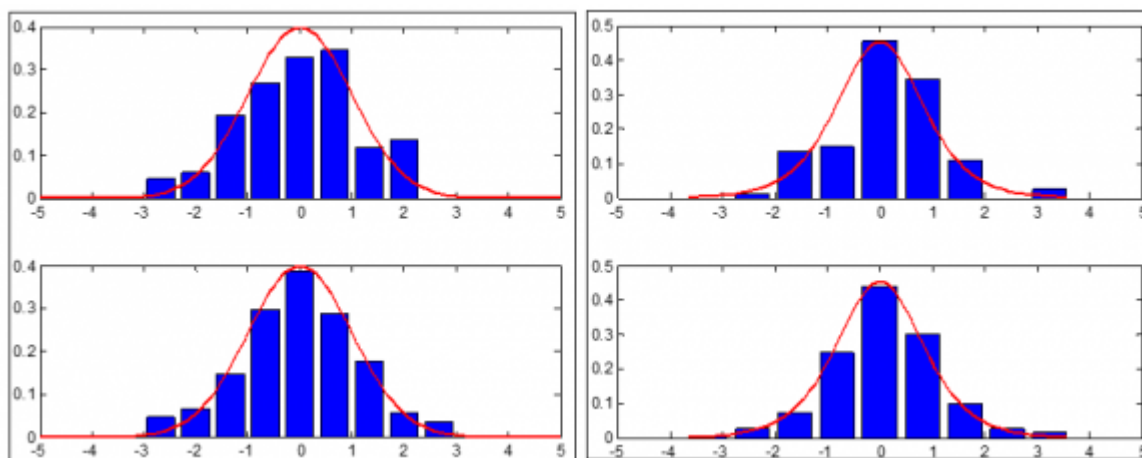
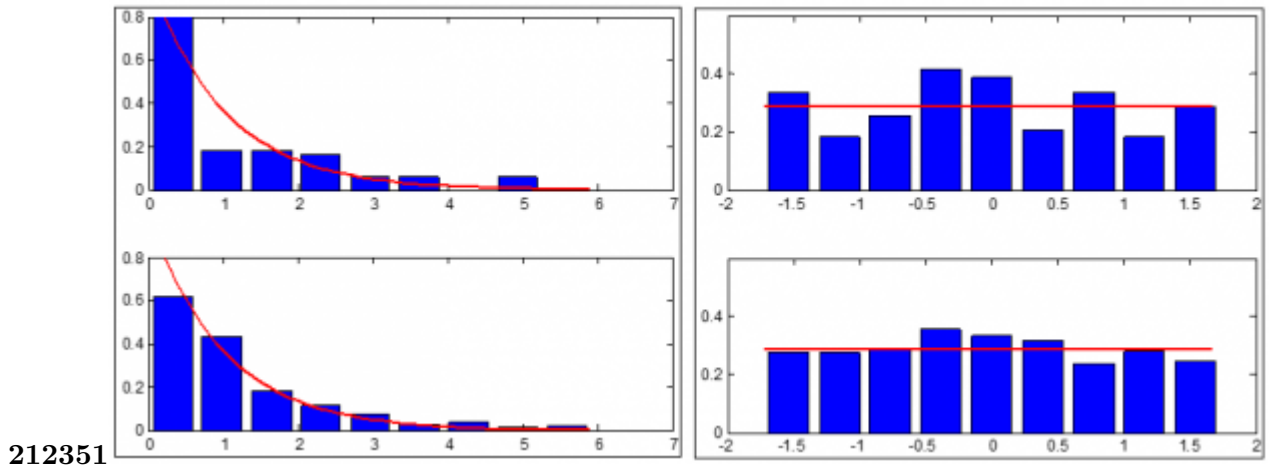
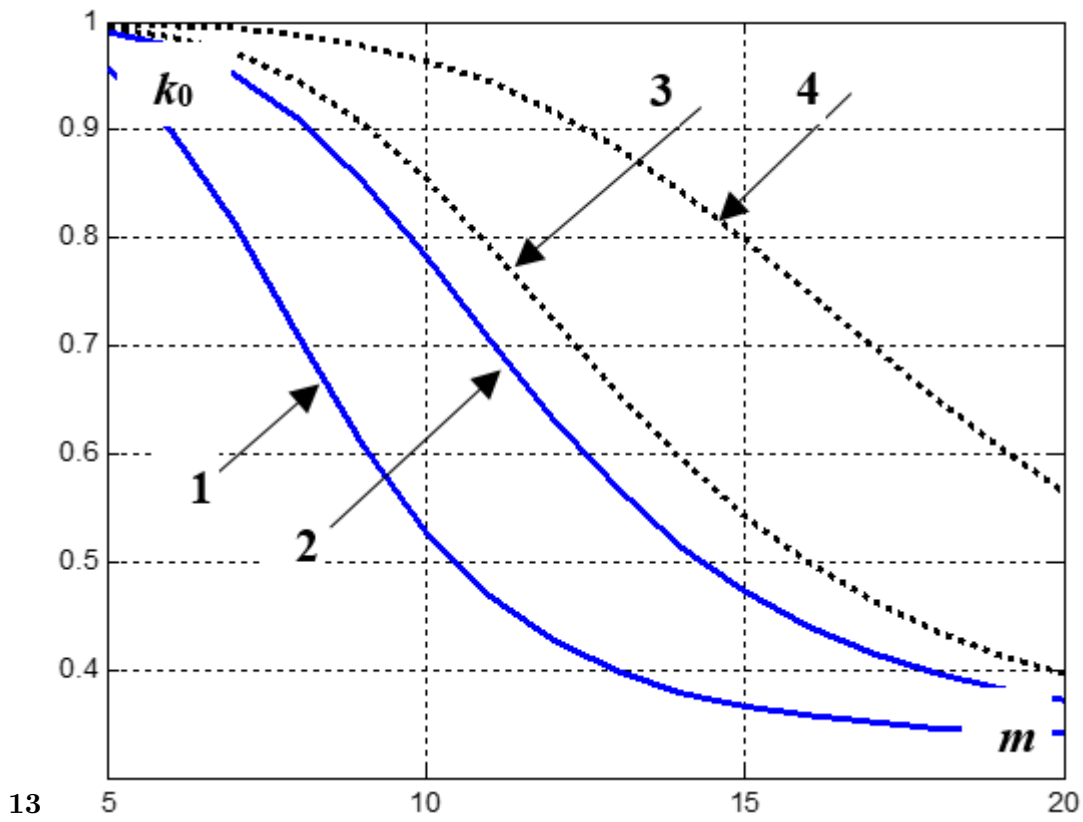


Figure 3: .



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Figure 4: 2 . 1 I ? ? 2 I ? . 3 . 5 . ??? . 1 :)



13

Figure 5:) -?????? 1 . 3 .,

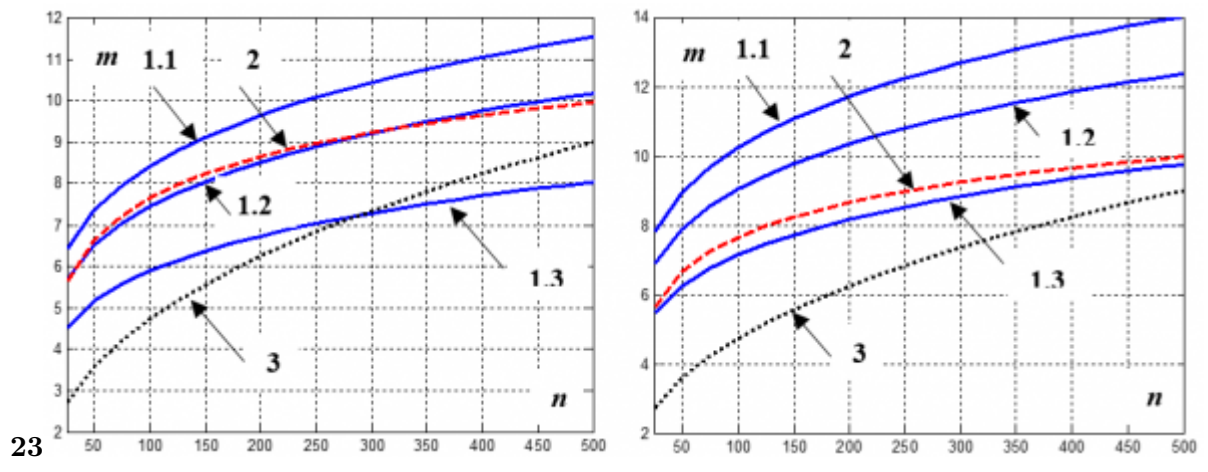


Figure 6: ???????? 2 :???. 3 :

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