CrossRef DOI of original article:

A Unique Method for Detecting and Characterizing Low Probability of Intercept Frequency Hopping Radar Signals by means of the Wigner-Ville Distribution and the Reassigned Smoothed Pseudo Wigner-Ville Distribution

Daniel L. Stevens

Received: 1 January 1970 Accepted: 1 January 1970 Published: 1 January 1970

Abstract

Low probability of intercept radar signals, which are may times difficult to detect and characterize, have as their goal ?to see but not be seen?. Digital intercept receivers are currently moving away from Fourier-based techniques and toward classical time-frequency techniques for analyzing low probability of intercept radar signals. This paper brings forth the unique approach of both detecting and characterizing low probability of intercept frequency hopping radar signals by employing and comparing the Wigner-Ville Distribution and the Reassigned Smoothed Pseudo Wigner-Ville Distribution. Four-component frequency hopping low probability of intercept radar signals were analyzed. The following metrics were used for evaluation: percent error of: carrier frequency, modulation bandwidth, modulation period, and time-frequency localization. Also used were: percent detection, lowest signal-to-noise ratio for signal detection, and relative processing time. Experimental results demonstrate that overall, the Reassigned Smoothed Pseudo Wigner-Ville Distribution produced more accurate characterization metrics than the Wigner-Ville Distribution. An improvement in performance could potentially translate into saved equipment and lives.

Index terms—

1 I. Introduction

probability of intercept radar signals, which are may times difficult to detect and characterize, have as their goal 'to see but not be seen'. Digital intercept receivers are currently moving away from Fourier-based techniques and toward classical time-frequency techniques for analyzing low probability of intercept radar signals. This paper brings forth the unique approach of both detecting and characterizing low probability of intercept frequency hopping radar signals by employing and comparing the Wigner-Ville Distribution and the Reassigned Smoothed Pseudo Wigner-Ville Distribution. Fourcomponent frequency hopping low probability of intercept radar signals were analyzed. The following metrics were used for evaluation: percent error of: carrier frequency, modulation bandwidth, modulation period, and time-frequency localization. Also used were: percent detection, lowest signalto-noise ratio for signal detection, and relative processing time. Experimental results demonstrate that overall, the Reassigned Smoothed Pseudo Wigner-Ville Distribution produced more accurate characterization metrics than the Wigner-Ville Distribution. An improvement in performance could potentially translate into saved equipment and lives. low probability of intercept (LPI) radar that uses frequency hopping techniques changes the transmitting frequency in time over a wide bandwidth to prevent an intercept receiver from intercepting the waveform. The frequency slots are chosen from a frequency hopping sequence, which is unknown to the intercept receiver, thereby giving the radar the advantage in processing gain over the intercept receiver. The frequency sequence appears random to the intercept receiver, thereby making it nearly impossible for the intercept receiver to follow the changes in frequency [PAC09]. This, in turn, prevents a jammer from jamming the transmitted frequency [ADA04]. Frequency hopping radar performance depends only slightly on the code used, given that certain properties are met. This allows for a larger assortment of codes, making it even more difficult to intercept.

Time-frequency signal analysis includes the analysis and processing of signals

2 a) Wigner-Ville Distribution (WVD)

One of the most prominent time-frequency distribution members is the WVD. The WVD satisfies a great number of desirable mathematical properties. It is always real-valued, it preserves time and frequency shifts, and it satisfies marginal properties [AUG96], [QIA02]. The WVD is a transformation of a continuous time signal into the time-frequency domain, and is computed by correlating the signal with a time and frequency translated version of itself, making the WVD bilinear. In addition, the WVD exhibits the highest signal energy concentration in the time-frequency plane **??**WIL06]. By using the WVD, an intercept receiver can come close to having a processing gain near the LPI radar's matched filter processing gain [PAC09]. The WVD also contains cross term interference between every pair of signal components, which may limit its applications [GUL07], [STE96], and which can make the WVD time-frequency representation hard to interpret, especially if the components are numerous or close to each other, and the more so in the presence of noise [BOA03]. This lack of readability can in turn translate into decreased signal detection and parameter extraction metrics, potentially placing the intercept receiver signal analyst in harm's way.

The WVD of a signal ??(??) is given in equation (1) as:?? ?? (??, ð ??"ð ??") = ? ??(?? + ?? 2 +? ??)?? * ??? ? ?? 2 ? ?? ??? 2??ð ??"ð ??"?? ????

or equivalently in equation (2) as:?? ?? (??, ð ??"ð ??") = ? ??(ð ??"ð ??" + ?? 2 +? ??)?? * ?ð ??"ð ??" ? ?? 2 ? ?? ?? 2?????? ???? b) Reassigned Smooth Pseudo Wigner-Ville Distribution (RSPWVD)

The original idea of reassignment was introduced in an attempt to improve the Spectrogram [OZD03]. As with any other bilinear energy distribution, the Spectrogram is faced with the trade-off between the reducing the misleading interference terms and sharpening the localization of the signal components.

We can define the Spectrogram as a twodimensional convolution of the WVD of the signal by the WVD of the analysis window, as in equation (3):?? ?? (??, ð ??"ð ??"; ?) = ? ?? ?? +? ?? (??, ??)?? ? (?? ? ??, ð ??"ð ??" ? ??)???? ????

Therefore, the distribution reduces the interference terms of the signal's WVD, but at the expense of time and frequency localization. But a closer look at equation (3) shows that ?? ? (?? ? ??, ð ??"ð ??" ? ??) delimits a time-frequency domain at the vicinity of the (??, ð ??"ð ??") point, inside which a weighted average of the signal's WVD values is performed. The key point of the reassignment principle is that these values really have no reason to be symmetrically distributed around (??, ð ??"ð ??"), the geometrical center of this domain. Their average should not be assigned at this point, but rather at the center of gravity of this domain, which is more representative of the local energy distribution of the signal [AUG94]. Using a mechanical analogy, the local energy distribution ?? ? (?? ? ??, ð ??"ð ??" ? ??)?? ?? (??, ??) (as a function of ?? ?????? ??) can be considered as a mass distribution, and it is much more accurate to assign the total mass (i.e. the Spectrogram value) to the center of gravity of the domain rather than to its geometrical center. Another way to look at it is this: the total mass of an object is assigned to its geometrical center, an arbitrary point which, except in the very specific case of a homogeneous distribution, has no reason to suit the actual distribution. A more meaningful choice is to assign the total mass of an object, as well as the Spectrogram value, to the center of gravity of their respective distribution [BOA03]. This is exactly how the reassignment method proceeds: it moves each value of the Spectrogram computed at any point (??, ð ??"ð ??") to another point (?? ,ð ??"ð ??" ?) which is the center of gravity of the signal energy distribution around (??, ð ??"ð ??") (see equations (**??**) and (**??**)) [LIX08]:?? (??; ??, ð ??"ð ??") = ? ?? ?? ? (?? ? ??, ð ??"ð ??" ? ??)?? ?? (??, ??)???? ???? +? ?? ? ?? ? (?? ? ??, ð ??"ð ??" ? ??)?? ?? (??, ??)???? ???? +? ?? ð ??"ð ??" ?(??; ??, ð ??"ð ??") = ? ?? ?? ? (?? ? ??, ð ??"ð ??" ? ??)?? ?? (??, ??)???? ???? +? ?? ? ?? ? (?? ? ??, ð ??"ð ??" ? ??)?? ?? (??, ??)???? ???? +? ??

leading to a reassigned Spectrogram (equation (**??**)), whose value at any point (?? ? , ð ??"ð ??" ?) is the sum of all the Spectrogram values reassigned to this point:?? ?? (??) (?? ? , ð ??"ð ??" ? ; ?) = ? ?? ?? +? ?? (??, ð ??"ð ??"; ?)????? ? ? ?? (??; ??, ð ??"ð ??")??? ?ð ??"ð ??" ? ? ð ??"ð ??" ?(??; ??, ð ??"ð ??")? ???? ??ð ??"ð ??"

An interesting property of this new distribution is that it also uses the phase information of the STFT, and not just its squared modulus, as in the Spectrogram. It uses this information from the phase spectrum in order to sharpen the amplitude estimates in both time and frequency. This can be seen from the following expressions of the reassignment operators: (??). This leads to an efficient implementation for the Reassigned Spectrogram without explicitly computing the partial derivatives of phase. The Reassigned Spectrogram may thus be computed by using 3 STFTs, each having a different window (the window function h; the same window with a weighted time ramp t*h; and, the derivative of the window function h with respect to time (dh/dt)). Reassigned Spectrograms are therefore very computationally efficient to implement.?? (??; ??, ð ??"ð ??") = ? ??? ?? (

Since time-frequency reassignment is not a bilinear operation, it does not permit a stable reconstruction of the signal. In addition, once the phase information has been used to reassign the amplitude coefficients, it is no longer available for use in reconstruction.

For this reason, the reassignment method has received limited attention from engineers, and its greatest potential seems to be where reconstruction is not necessary, that is, where signal analysis is an end unto itself.

One of the most important properties of the reassignment method is that the application of the reassignment process to any distribution of Cohen's class, theoretically yields perfectly localized distributions for chirp signals, frequency tones, and impulses. This is one of the reasons that the reassignment method was chosen for this paper as a signal processing technique for analyzing LPI radar waveforms such as the frequency hopping waveforms (which can be viewed as multiple tones).

In order to resolve the classical time-frequency analysis deficiency of cross-term interference, a method needs to be used which reduces cross-terms, which the reassignment method does.

The reassignment principle for the Spectrogram allows for a straight-forward extension of its use for other distributions as well [HIP00], including the WVD. If we consider the general expression of a distribution of the Cohen's class as a two-dimensional convolution of the WVD, as in equation (**??**1

3 II. Methodology

The methodologies detailed in this section describe the processes involved in obtaining and comparing metrics between the classical time-frequency analysis techniques of the Wigner-Ville Distribution and the Reassigned Smoothed Pseudo Wigner-Ville Distribution for the detection and characterization of low probability of intercept frequency hopping radar signals.

The tools used for this testing were: MATLAB (version 8.3), Signal Processing Toolbox (version 6.21), and Time-Frequency Toolbox (version 1.0). All testing was accomplished on a desktop computer.

Testing was performed for the 4-component frequency hopping waveform. Waveform parameters were chosen for academic validation of signal processing techniques. Due to computer processing resources they were not meant to represent real-world values. The number of samples for each test was chosen to be 512, which seemed to be the optimum size for the desktop computer. Testing was performed at three different SNR levels: 10dB, 0dB, and the lowest SNR at which the signal could be detected. The noise added was white Gaussian noise, which best reflects the thermal noise present in the IF section of an intercept receiver [PAC09]. Kaiser windowing was used, when windowing was applicable. 100 runs were performed for each test, for statistical purposes. The plots included in this paper were done at a threshold of The frequency hopping (prevalent in the LPI arena [AMS09]) 4-component signal had parameters of: sampling frequency=5KHz; carrier frequencies=1KHz, 1.75KHz, 0.75KHz, 1.25KHz; modulation bandwidth=1KHz; modulation period=.025sec.

After each particular run of each test, metrics were extracted from the time-frequency representation.

The different metrics extracted were as follows:

1) Relative Processing Time: The relative processing time for each time-frequency representation.

2) Percent Detection: Percent of time signal was detected. Signal was declared a detection if any portion of each of the 4 signal components exceeded a set threshold (a certain percentage of the maximum intensity of the time-frequency representation). Threshold percentages were determined based on visual detections of low SNR signals (lowest SNR at which the signal could be visually detected in the timefrequency representation). Based on the above methodology, thresholds were assigned as follows for the signal processing techniques used for this paper: WVD (50%); RSPWVD (50%).

For percent detection determination, these threshold values were included in the time-frequency plot algorithms so that the thresholds could be applied automatically during the plotting process. From the threshold plot, the signal was declared a detection if any portion of each of the signal components was visible (see Figure 1). The threshold percentage was determined based on manual measurement of the modulation bandwidth of the signal in the time-frequency representation. This was accomplished for ten test runs of each time-frequency analysis tool (WVD and RSPWVD). During each manual measurement, the max intensity of the high and low measuring points was recorded. The average of the max intensity values for these test runs was 20%. This was adopted as the threshold value, and is representative of what is obtained when performing manual measurements. This 20% threshold was also adapted for determining the modulation period and the time-frequency localization (both are described below).

For modulation bandwidth determination, the 20% threshold value was included in the time-frequency plot algorithms so that the threshold could be applied automatically during the plotting process. From the threshold plot, the modulation bandwidth was manually measured (see Figure 3). For lowest detectable SNR determination, these threshold values (WVD (50%); RSPWVD (50%)) were included in the time-frequency plot algorithms so that the thresholds could be applied automatically during the plotting process. From the threshold plot, the signal was declared a detection if any portion of each of the 4 signal components was visible. The lowest SNR level for which the signal was declared a detection is the lowest detectable SNR.

The data from all 100 runs for each test was used to produce the actual, error, and percent error for each of these metrics listed above.

The metrics from the WVD were then compared to the metrics from the RSPWVD. By and large, the RSPWVD outperformed the WVD, as will be shown in the results section.

4 III. Results

Table 1 presents the overall test metrics for the two classical time-frequency analysis techniques used in this testing (WVD versus RSPWVD). 1, the RSPWVD outperformed the WVD in average percent error: carrier frequency (0.12% vs. 0.21%), modulation bandwidth (4.72% vs. 6.07%), modulation period (6.05% vs. 16.51%), and timefrequency localization (y-direction) (1.28% vs. 2.14%); and in average: percent detection (94.1% vs. 90.2%), lowest detectable SNR (-3.0dB vs. -2.0dB) and average relative processing time (0.023s vs. 0.682s).

Figure 6 shows comparative plots of the WVD vs. the RSPWVD (4-component frequency hopping) at SNRs of 10dB (top), 0dB (middle), and lowest detectable SNR (-2.0dB for WVD and -3.0dB for RSPWVD) (bottom).

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6 IV. Discussion

This section will elaborate on the results from the previous section.

From Table 1, the RSPWVD outperformed the WVD in average percent error: carrier frequency (0.12% vs. 0.21%), modulation bandwidth (4.72% vs. 6.07%), modulation period (6.05% vs. 16.51%), and timefrequency localization (y-direction) (1.28% vs. 2.14%); and in average: percent detection (94.1% vs. 90.2%), lowest detectable SNR (-3.0dB vs. -2.0dB) and average relative processing time (0.023s vs. 0.682s). These results are the result of the RSPWVD signal being a more localized signal than the WVD signal, along with the fact that the WVD signal has cross-term interference, which the RSPWVD doesn't have.

The RSPWVD might be used in a scenario where you need good signal localization in a fairly low SNR environment, in a short amount of time. The RSPWVD would be preferred over the WVD in virtually every scenario, based on the metrics obtained. Digital intercept receivers, whose main job is to detect and extract parameters from low probability of intercept radar signals, are currently moving away from Fourier-based analysis and moving towards classical time-frequency analysis techniques, such as the WVD and the RSPWVD, for the purpose of analyzing low probability of intercept radar signals. Based on the research performed for this paper (the novel direct comparison of the WVD versus the RSPWVD for the signal analysis of low probability of intercept frequency hopping radar signals) it was shown that the RSPWVD by and large outperformed the WVD for analyzing these low probability of intercept radar signals -for reasons brought out in the discussion section above. More accurate characterization metrics may well equate to saved equipment and lives.

Future plans include analysis of an additional low probability of intercept radar waveform 8-component frequency Hopper, again using the WVD and the RSPWVD as time-frequency analysis techniques.

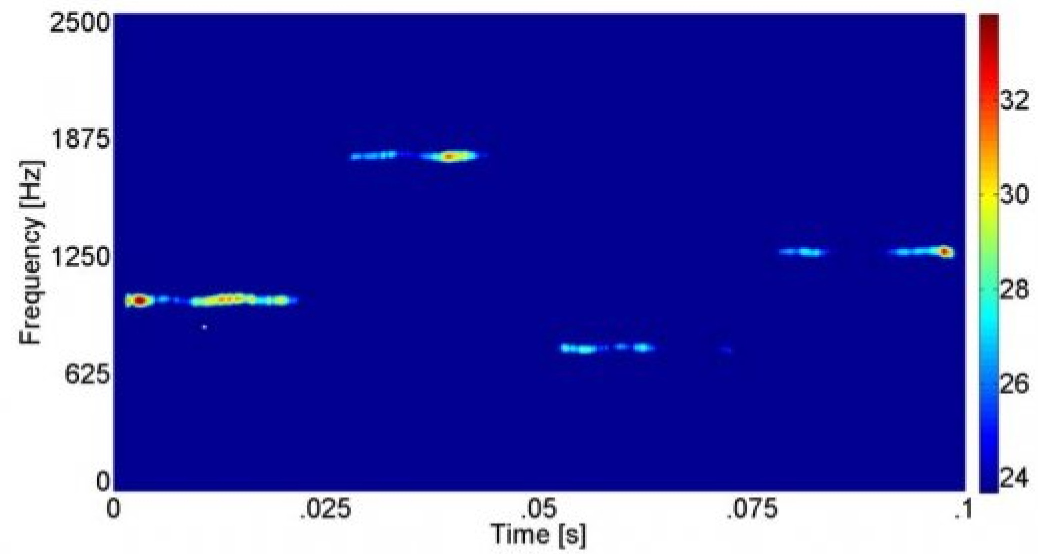

Figure 1:

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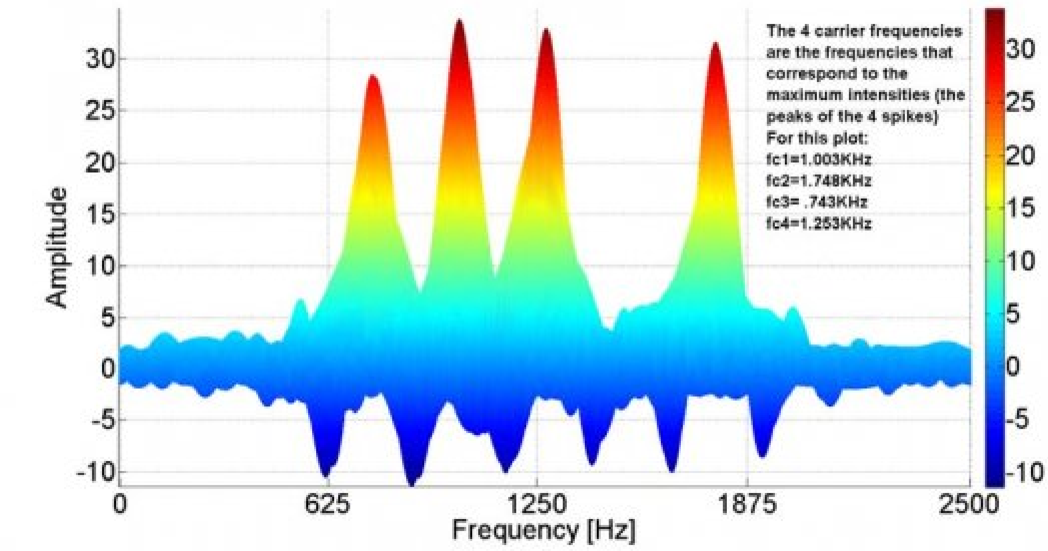

Figure 2:

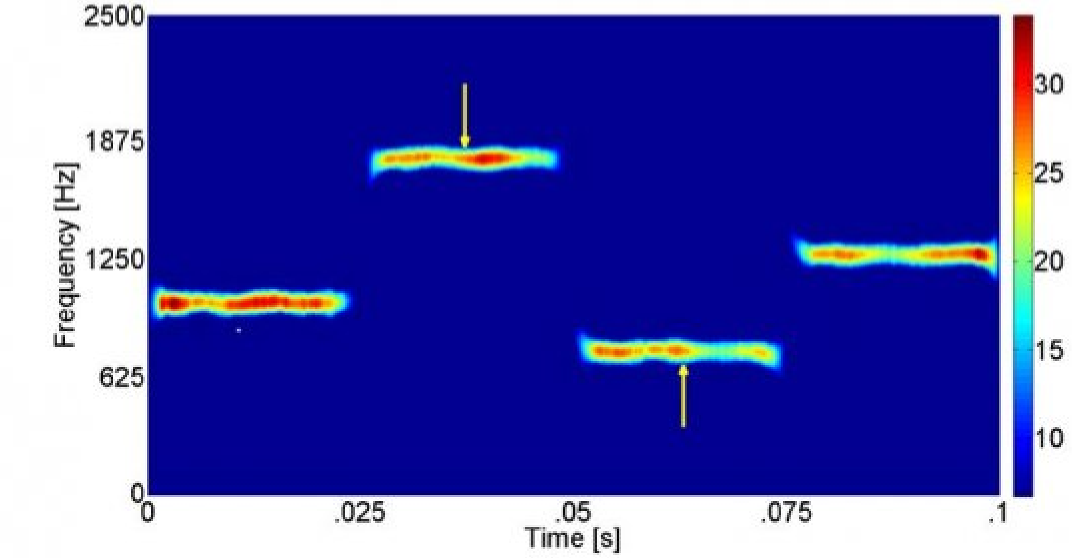

Figure 3:

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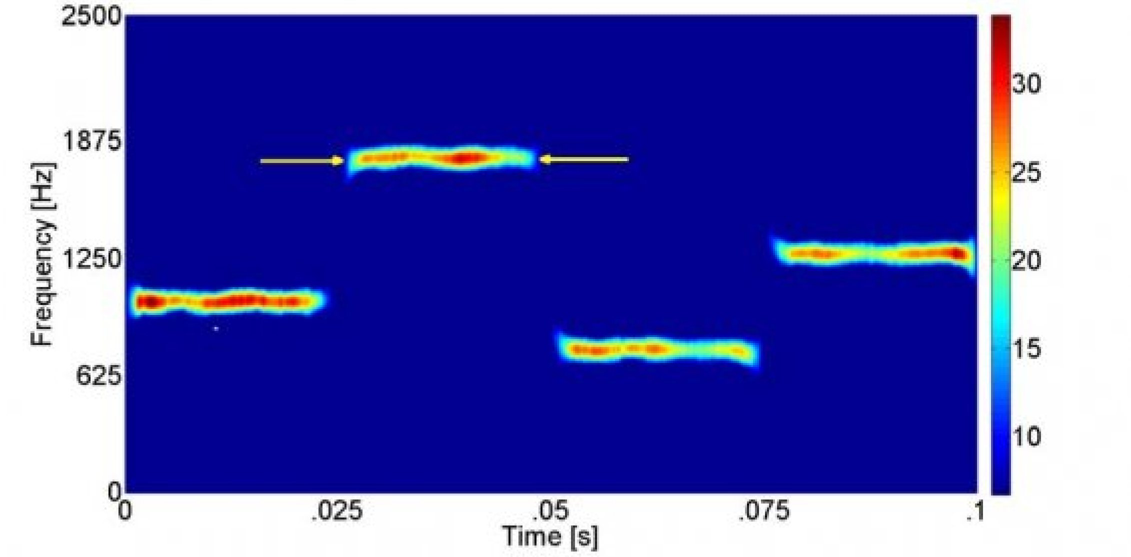

Figure 4: Figure 1 :

2

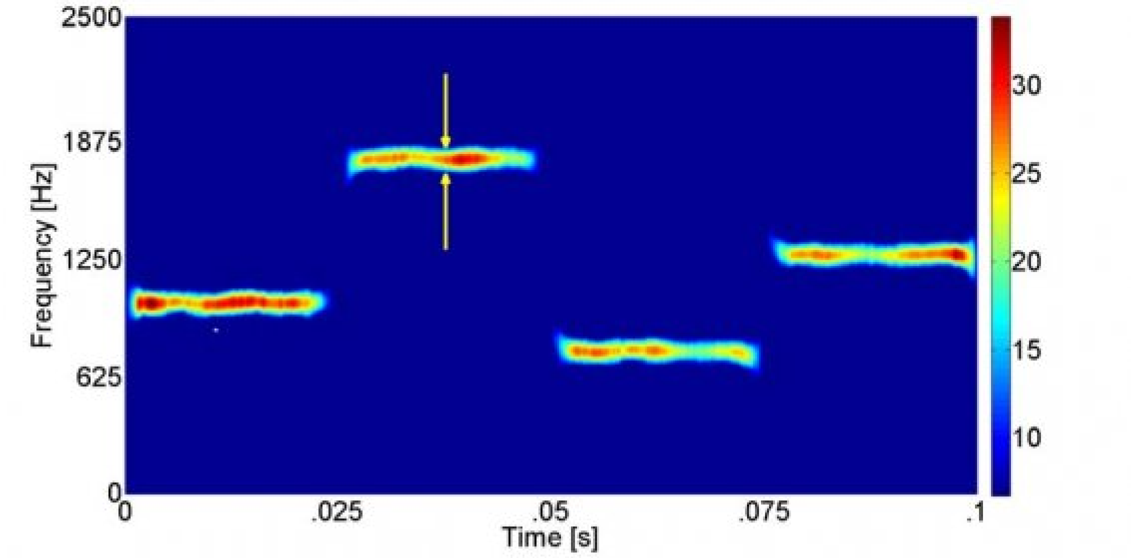

Figure 5: Figure 2 :

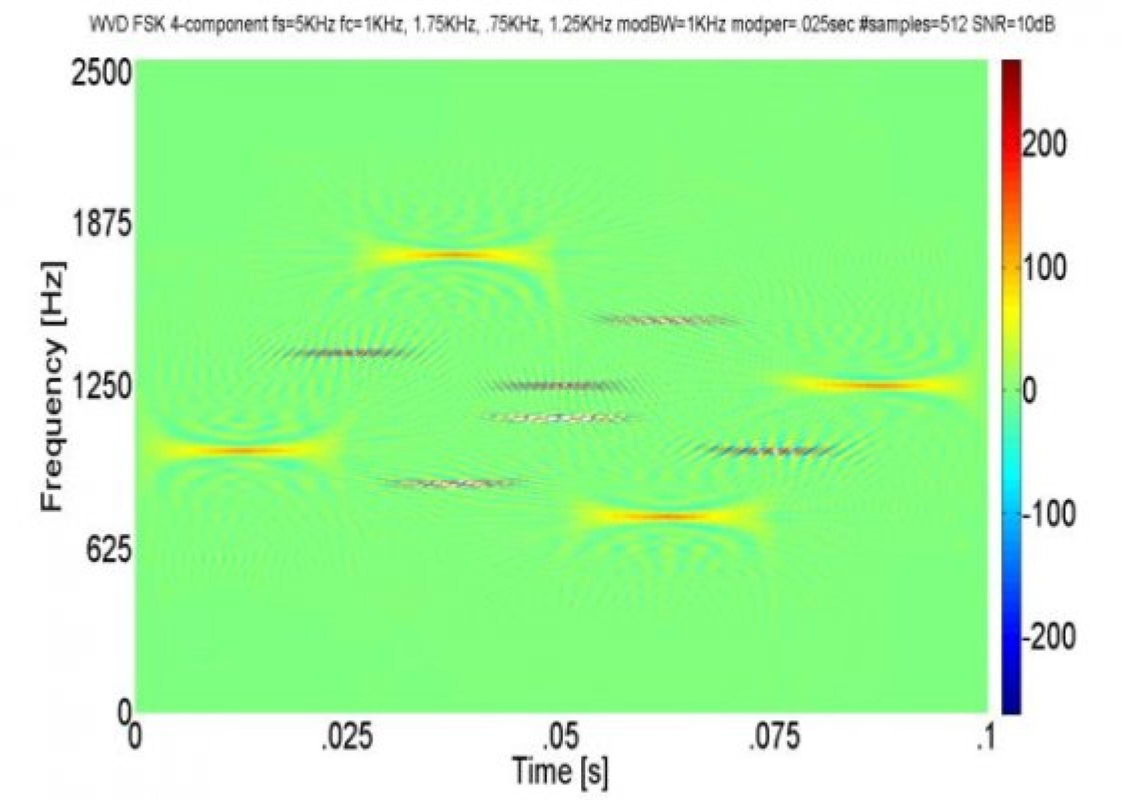

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Figure 6: Figure 3 :FA

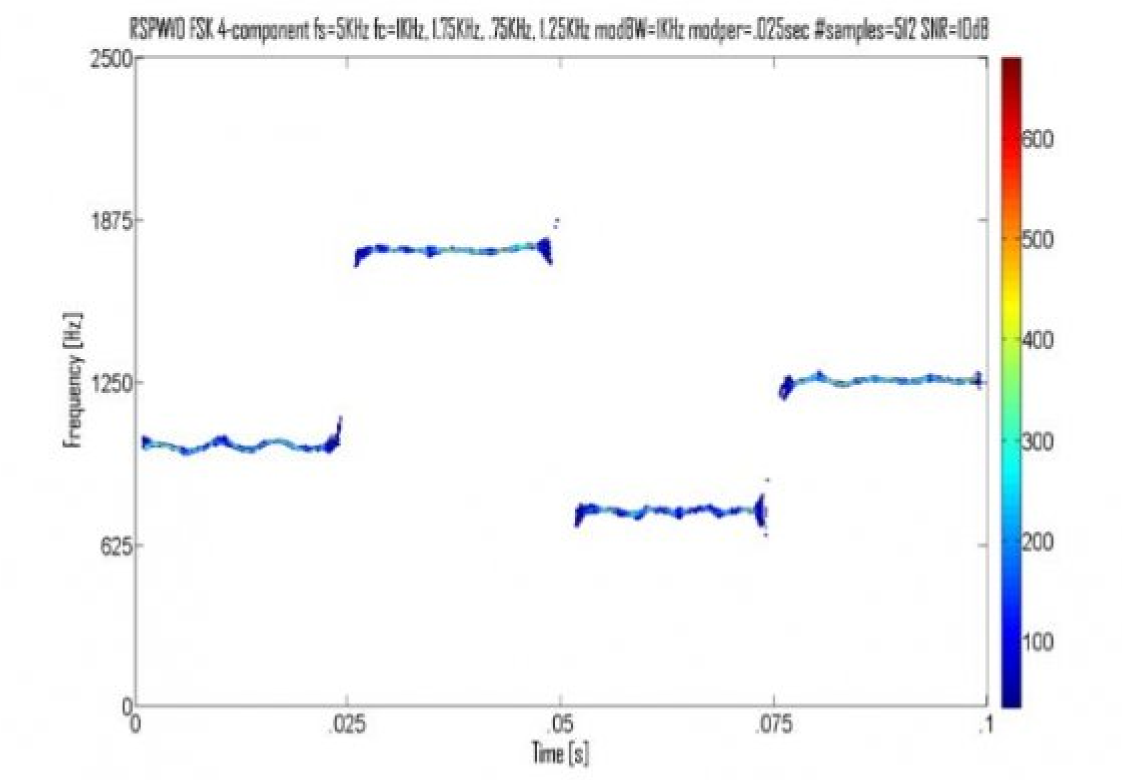

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Figure 7: Figure 4 :

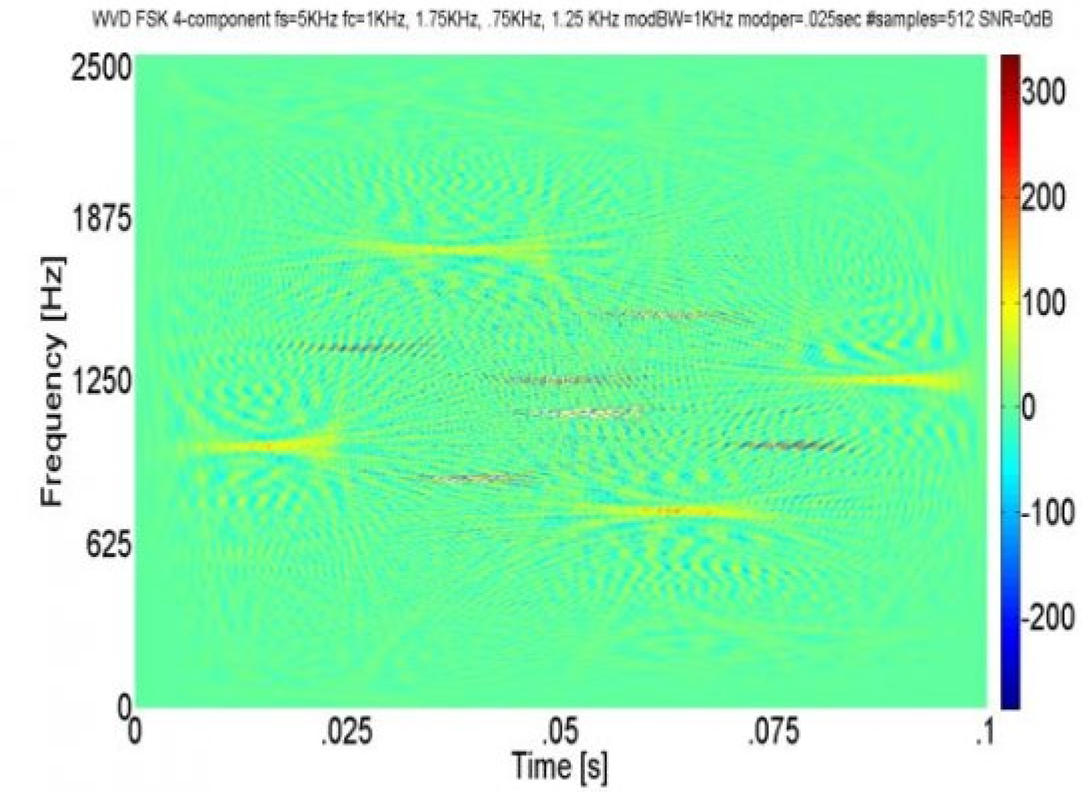

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Figure 8: Figure 5 :

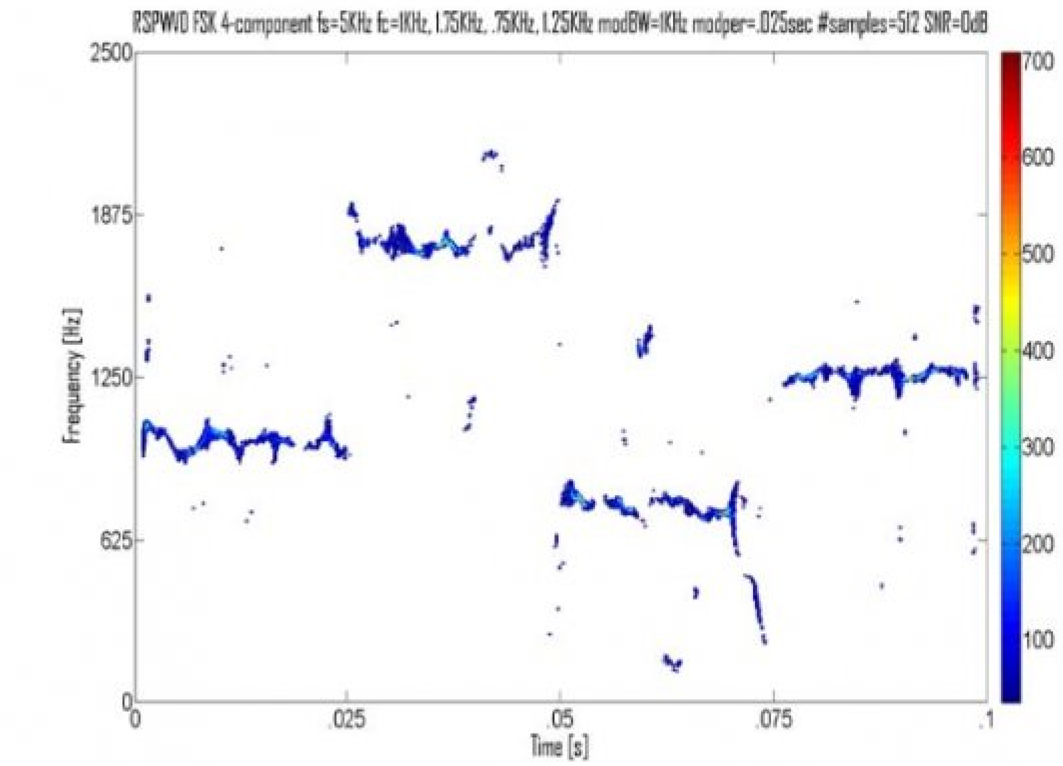

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Figure 9: Figure 6 :

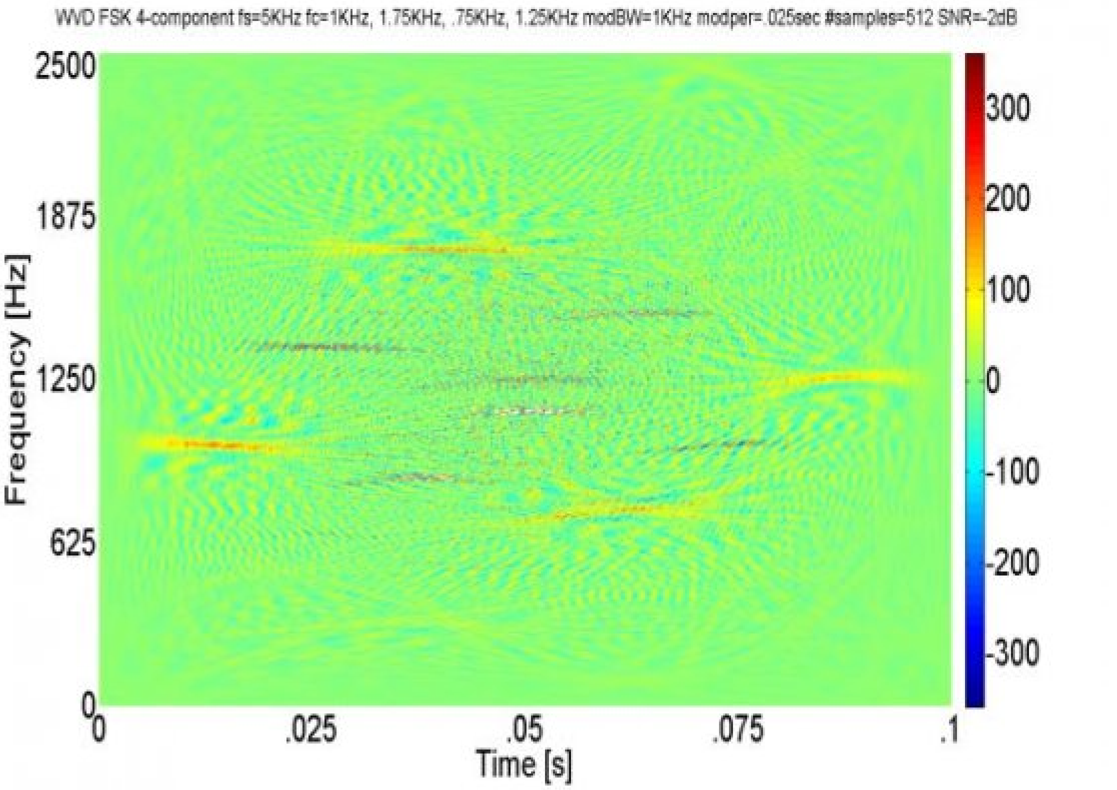

Figure 10:

1

carrier frequency, modulation bandwidth, modulation period; average: time-frequency localization-y (as percent of y-axis), percent detection, lowest detectable snr, relative processing time) for the two classical time-frequency analysis techniques (WVD versus RSPWVD)

Parameters	WVD	RSPWVD
Carrier Frequency	0.21%	0.12%
Modulation Bandwidth	6.07%	4.72%
Modulation Period	16.51%	6.05%
Time-Frequency Localization-Y	2.14%	1.28%
Percent Detection	90.2%	94.1%
Lowest Detectable SNR	-2.0dB	-3.0dB
Relative Processing Time	0.682s	0.023s

From Table

Figure 11: Table 1 :

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Figure 12: F

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