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Strictly as per the compliance and regulations of:
Peculiarities of UAV Construction for Flights at High Altitudes

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Abstract - A mathematical model of UAV power supply system is developed, consisting of a set of solar cells, a set of electric batteries and a subsystem of transmission and distribution of electricity. The mathematical apparatus was based on the indicators of solar radiation power, the level of battery charge, the efficiency of electric batteries and solar cells, and geographical coordinates as well as the height of the aircraft position. In this case, the basic model was improved through the inclusion of algorithms for the analysis of losses of electric batteries and averaging the function of solar radiation power by time in accordance with the set of time ranges. The mathematical apparatus for calculation and optimization of aerodynamic parameters of UAV was also suggested on the basis of such indicators as the coefficient of lifting force, drag coefficient, air density as a function of flight altitude and total wing area of the aircraft. The suggested methods of analysis of structural features of HALE-class UAV provide an opportunity to both build automated evaluation algorithms and to carry out the procedure of expert evaluation of aircraft through the analysis of the corresponding dependencies: the ratio of the lifting force factor on the angle of attack and the drag coefficient, the ratio of the lifting force and aerodynamic resistance as a function of the drag coefficient, the dependence of the specific power on the load on the wing produced by the height of the UAV aircraft.

I. Introduction

The active introduction of unmanned aerial vehicles (UAVs), observed during the last decade [1, 2], significantly expanded the instrumental basis of optical [3], chemical [4] and radio monitoring systems [5], and also provided fundamentally new opportunities for the organization of cargo transportation [6] and telecommunication systems [7] (Fig.1). At the same time, as noted by the researchers [8-10], the development of unmanned aviation systems of long-range action for flights at high altitudes will assist in covering with the help of UAV the important domains of satellite communications and satellite imagery, which today are characterized by extremely high estimate associated with the need to bring artificial satellites into the Earth’s orbit.

Figure 1: Areas of application of modern unmanned aviation systems

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This type of devices capable of non-stop staying at the level of tropopause and stratosphere for a long period of time, belongs to the class “HALE” (High-Altitude Long Endurance). To construct such devices, it is necessary to develop a power system of the device capable of functioning without providing fuel and maintenance, as well as to take into account the features of UAV flight at altitudes from 12-16 km above sea level (depending on the latitude, the indicator of which determines the level of tropopause). It should be noted that nowadays numerous projects on construction of HALE-class UAVs, among which ERAST [11], HALE UAV [12], Helios [13], HeliPLA [14] and Zephyr [15] are worth mentioning, did not show the desired results, which indicates the relevance of this study.

Analysis of scientific publications on this subject included papers that provide fairly simple and adequate models of calculation of average values of intensity of solar radiation and angle of incidence of sunlight [16, 17] as target functions from arguments of longitude, latitude and height of UAV dislocation, as well as time (time of year and time of day). Further, methods of optimization of the power supply system, which includes a system of solar cells and a set of electric batteries, which can be effectively applied in the construction of HALE-class UAV [17, 18], were determined. Finally, the results of practical research on the determination of dependence of change of UAV aerodynamic parameters on its height above the sea level [17-20] were considered. The analysis indicated the unsolved part of the general task in the field of construction of HALE-class UAV construction and was taken as the basis for further research. The purpose of this paper, thus was to develop a methodological base for improving the design of long-range UAV for flights at high altitudes, on the basis of which it would possible to construct algorithms for calculating the optimal parameters of these devices and provide appropriate guidelines for developers.

II. Methodology of the Study

As mentioned above, the basic classification of key elements, the peculiarities of which should be analysed in the construction of a mathematical model of HALE-class UAV, includes a power supply system. Power supply of this type of UAV is usually based on a system of solar cells located on the surface of the apparatus, which, due to the photoelectric effect, convert light energy into electrical energy. Another element of the power supply system is a set of electric batteries that are charged from solar panels at sufficient intensity of solar radiation and, in turn, ensure the operation of the UAV at a time when the intensity of solar radiation is insufficient (in general, the time interval can be divided into day and night time). The mathematical model, respectively, will include both the parameters of these subsystems, and the algorithm of optimal distribution of electricity, as well as the peculiarities of UAV functioning and factors related to the external environment that affect the level of electricity consumption (Fig. 2).

![Basic scheme of organization of power supply system of HALE-class UAV](image)

**Figure 2:** Basic scheme of organization of power supply system of HALE-class UAV

Thus, the mathematical model of the HALE-class UAV power system consists of the following elements:

- The power of solar radiation as a function of time $P_s(t)$, which can be averaged for certain periods of
time \( \int_{t_1}^{t_2} P_S(t)\, dt/(t_2 - t_1) \) and determined in accordance with climatic conditions and seasons;

- The level of the battery charge \( P_{CH} \);
- Performance indicators of electric batteries \( \eta_{SB} \) (storage battery, SB) and solar panels \( \eta_{SC} \) (storage cell, SC), as well the coefficient of transmission losses in the system \( k_{TR} \) (transmittance factor, TR);
- Geographic coordinates of UAV: longitude \( \lambda \) and latitude \( \varphi \), as well as the hourly angle, as a function of time \( \omega_S(t) \), determining the position of the Sun.

On the basis of these values and functions, it is possible to determine the optimal values for \( \tau \) the moment of switching to the battery charging mode and the moment of \( \tau' \) switching to the battery usage mode according to the value of solar radiation power \( P_0 = P_S(\tau) = P_S(\tau') \) and, respectively, to organize the distribution of the electric power within the system of UAV power supply.

Another important task is to simulate UAV flight and determine its aerodynamic parameters depending on the height of the device dislocation above the sea level. Obviously, when solving the optimization problem, the key arguments of the target functions will be the lifting force \( Y \) and aerodynamic resistance \( X \), which, in turn, are determined on the basis of the following values:

- Coefficient of lifting force \( C_Y \);
- Drag coefficient \( C_X \);
- Air density as a function of UAV flight altitude — \( \rho(h_{UAV}) \);
- Total wing area of UAV — \( S_W \);
- The relative speed of the UAV — \( v_{UAV} \).

In this case, the value of the lifting force coefficient and the drag coefficient depends on the aerodynamic profile of the UAV, the angle of attack and the Reynolds number — \( Re \).

On the basis of the suggested approach, it is possible to calculate and compare with experimental data the dependences for aerodynamic quality of UAV as the ratio of lifting force to aerodynamic resistance, specific power consumed by UAV in accordance with the flight mode, bearing capacity of UAV according to the load on the wing, etc.

### III. Results of the Study

The analysis, which was carried out in the previous section, indicated that in order to calculate the optimal moment of switching between the mode of charging batteries and the mode of using batteries, it is necessary to average the function of solar radiation power by time. If at the level of the basic model we put the value \( \eta_{SB} = 1 \) (that is, we completely neglect the losses in the battery system), then the values \( \tau \) and \( \tau' \) can be determined through the solution of the following equation with respect to the variable \( t \):

\[
\tau = \tau' = t \text{ для } P_S(t) \times \frac{\int_{t_0}^{t_{24}} P_S(t)\, dt}{\Delta t} \quad \text{при } \eta_{SB} = 0.
\]

where \( \Delta t = t_{24} - t_{00} \) corresponds to a time interval covering one full day. Fig. 3 shows such a solution as the determination of the coordinates of \( P_S(t) \) and \( P_S'(t) \) functions intersection.

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**Figure 3:** Algorithm for calculating the switching moment between the mode of charging batteries and the mode of using batteries.
However, the construction of an adequate mathematical model necessitates the analysis of losses of electric batteries, and therefore the consideration of $P(t)$ as a complex function, the value of which depends on $\eta_{SB}$ (as shown in Figure 3, the dependence is close to linear). In the framework of this study we would also like to consider the simplest situation, when the function of solar radiation power depends entirely on the position of the Sun (the case of cloudless weather or flight at extremely high altitudes). Accordingly, time range $\Delta t$ can be divided into 4 time diapasons (Fig. 3, generalized dependencies $P_3(t)$ and $P_3(\eta_{SB})$ are averaging statistical data of works [16, 17]):

$$\int_{t_{SR}}^{t_{SS}} \left( \frac{P_0 - GSC \cdot k_{TR} \cdot \eta_{SC} \cdot \sin(\alpha(t))}{\eta_{SB}} \right) dt,$$

(4)

Correlation of the coordinates of the UAV and the hour angle of the Sun provides an opportunity to determine the angle of incidence of sunlight on the surface of the solar cell:

$$\alpha = \frac{\pi}{2} - \cos^{-1} \left( \cos(\varphi) \cdot \cos(\lambda) \cdot \cos(\omega_S(t)) + \sin(\varphi) \cdot \sin(\lambda) \right),$$

(2)

on the basis of which the optimal value is calculated $P_0$:

$$P_0 = GSC \cdot k_{TR} \cdot \eta_{SC} \cdot \sin(\alpha(t)),$$

(3)

where $GSC$ is the solar constant, SC, as the value of the total flux of solar radiation, per unit time and per unit area, oriented perpendicular to the flow. Accordingly, the UAV battery capacity is calculated on the basis of $P_0$ taking into account the time intervals:

$$P_{CH} = \int_{t_{SR}}^{t_{SS}} \left( \frac{P_0 - GSC \cdot k_{TR} \cdot \eta_{SC} \cdot \sin(\alpha(t))}{\eta_{SB}} \right) dt.$$

(4)

Considering that within the framework of the model the equality of $\Delta t_A = \Delta t_E$ and $\Delta t_B = \Delta t_D$ gaps is accepted and the system of equations (4) can be simplified:

$$P_{CH} = 2 \frac{P_0 \cdot \Delta t_A}{\eta_{SB}} + \int_{t_{SR}}^{t} \left( \frac{2(P_0 - GSC \cdot k_{TR} \cdot \eta_{SC} \cdot \sin(\alpha(t)))}{\eta_{SB}} \right) dt.$$

(5)

Similarly, it is enough to calculate $P(t)$ for three time ranges:

$$\begin{align*}
\bar{P}(t) &= \frac{P_0}{\eta_{SB}} \text{ для } t \in [t_{00}; t_{SS}] \cup [t_{SS}; t_{24}] \\
\bar{P}(t) &= \frac{P_0 - P_3(t)}{\eta_{SB}} \text{ для } t \in [t_{SR}; t] \cup [t; t_{SS}] \\
\bar{P}(t) &= P_0 \text{ для } t \in [t; t']
\end{align*}$$

(6)

The suggested mathematical apparatus helps to determine areas (according to climatic zones), where HALE-class UAV can function autonomously for a sufficiently large period of time. When conducting cartographic research, it is sufficient to apply isomorphic lines calibrated according to the minimum permissible level $\bar{P}(t)$.

The next stage is calculation and optimization of aerodynamic parameters of UAV. As it was indicated in the previous section, the basis of mathematical
modelling at this stage is the parameters of lifting force and aerodynamic resistance:

\[
\begin{align*}
Y &= C_Y \cdot \rho(h_{UAV}) \cdot \left(\frac{v_{UAV}}{C_Y}\right)^2 \cdot S_W \quad \Rightarrow \quad Y = \frac{C_Y \cdot X}{C_Y} \\
X &= C_X \cdot \rho(h_{UAV}) \cdot \left(\frac{v_{UAV}}{C_Y}\right)^2 \cdot S_W
\end{align*}
\]  

When calculating the lifting force of UAV aerodynamic profile of the wing is suggested to be modelled at the level of a two-dimensional problem. At the same time, the calculation of \( C_X \) is more non-trivial, for this purpose it is necessary to determine the drag coefficient \( C_{AD} \) (airfoil drag, AD), which can also be calculated for a two-dimensional model of the aerodynamic profile, the coefficient responsible for the component of the parasitic resistance \( C_{PD} \) (parasitic drag, PD) and is determined experimentally, as well as inductive resistance \( C_{ID} \) (induced drag, ID). In this case, the inductive resistance is calculated on the basis of the coefficient of lifting force and the indicator of the relative elongation of the wing \( k_{AR} \) (aspect ratio, AR). Based on these indicators, the drag coefficient is calculated as the sum:

\[
P_{LF} = T \cdot v_{UAV} \quad \Rightarrow \quad P_{LF} = \frac{Y}{C_Y} \cdot v_{UAV} \quad \Rightarrow \quad P_{LF} = C_X \cdot \frac{2}{S_W \cdot \rho(h_{UAV})} \cdot \frac{m_{UAV} \cdot g(h_{UAV})}{C_Y}^{3/2}. \tag{10}
\]

Accordingly, the power density (power density, PD) for horizontal flight can be calculated as a ratio \( P_{LF} \) and \( S_{UAV} \):

\[
P_{PD}^{LF} = C_X \cdot \frac{2}{\rho(h_{UAV})} \cdot \frac{m_{UAV} \cdot g(h_{UAV})}{S_W}^{3/2}. \tag{11}
\]

It should be noted that dependence \( P_{PD}^{LF} \sim (\rho(h_{UAV})^{-1/2}) \) allows to construct dependence of load on a wing \( P_{PD}^{DF} \) in accordance with specific design peculiarities of UAV for different heights of horizontal flight. The indicator characterizing the HALE-class UAV in accordance with the possibilities of its optimization is the coefficient of bearing capacity \( C_M \), calculated as the ratio of mass of the bulk of the UAV (constant mass) — \( M_C \) and the mass of the elements to be optimized during the revision of the device, i. e. wings and solar panels that cover them - \( M_W \) (total mass \( M_Z = M_C + M_W \)). Thus it is expedient to define \( M_W \) as product of index of density \( \rho_W \) and total area of wing of UAV:

\[
C_M = \frac{M_Z - \rho_W \cdot S_W}{M_Z}. \tag{12}
\]

where the coefficient \( k_e \) is based on the Oswald coefficient of efficiency for HALE-class UAV \( k_e \approx \pi \). Adequacy of the suggested mathematical apparatus and its improvement in accordance with the specific task of UAV construction should be further checked on the basis of experimental tests, such as through the use of an aerodynamic tunnel.

Further work on modelling and optimization of UAV predetermines the introduction of the concept of thrust of the aircraft \( T \), and determining its speed on the basis of the total mass value \( m_{UAV} \):

\[
v_{UAV} = 2 \sqrt{\frac{(m_{UAV} \cdot g(h_{UAV}))}{(\rho(h_{UAV}) \cdot S_W \cdot C_Y)}}. \tag{9}
\]

The value of the UAV speed can be taken as the basis for calculating the power used in the horizontal mode of flight of the device (level flight, LF):

\[
P_{LF} = \frac{Y}{C_Y} \cdot v_{UAV} \Rightarrow \quad P_{LF} = C_X \cdot \frac{2}{S_W \cdot \rho(h_{UAV})} \cdot \frac{m_{UAV} \cdot g(h_{UAV})}{C_Y}^{3/2} \tag{10}
\]

As shown by the results of practical studies, for successful series of UAVs, the indicator \( C_M \) represents a fixed value [21-24] lying within the limits \( C_M \in [0.15; 0.22] \).

It should be noted that this approach is basic, and in solving the practical problem of developing a HALE-class UAV, the classification of components of the apparatus according to the extent to which these components are subject to optimization should be more detailed.

IV. Analysis of the Results

The conducted research on definition of principles of construction of long-range UAV for flights at high altitudes provided an opportunity to develop a methodological base that can be applied during the analysis of structural peculiarities of aircraft of the specified class in order to optimize them in accordance with the goals set. The work included modelling of the power supply system and aerodynamic parameters of the UAV. In this section, on the basis of the constructed models, we are going to suggest to develop a scheme of complex analysis, which can be used by developers in the construction and optimization of corresponding aircraft.
This scheme includes the following stages (Fig. 4):

- Determination of the conditions of the problem put before the developers of UAV: the desired values of the altitude of flight and the time of autonomous functioning of the device, as well as the amount of additional load that is planned to be used in the process of work, climatic conditions of the region, respectively, the average and minimum values of the solar irradiation per day depending on the season, limitation of the mass and dimensions of the device, the overall estimate of the project, etc.;
- Classification of components of the base model of UAV in accordance with the functional purpose, weight and possibility of further modification or replacement in the process of model improvement;
- Determination of total weight and total area of UAV wings;
- Classification of elements of the power supply system, which includes such groups as solar cells, electric batteries and subsystem of transmission and distribution of electricity;
- Determination of parameters of components of a power supply system and construction of basic algorithm of switching between modes of charging the batteries and usage of the batteries;
- Derivation of equations for calculating the level of specific power of consumption of electricity used in...
the horizontal mode of flight of the device (depending on the task, losses on manoeuvres, payload, power supply of the flight control system can be calculated, etc.);

- Iterative process of finding optimal parameters of UAV, which is carried out at the level of establishing the extremes of target functions, determined according to the conditions of the task of UAV construction of UAV;
- Specification of initial conditions and basic model if necessary.

It should also be noted that at the level of expert evaluation it is important to analyse the dependencies that can be obtained during the implementation of the above stages: the dependence of the lifting force factor on the angle of attack and the drag coefficient, the ratio of the lifting force and aerodynamic resistance as function on the drag coefficient, the dependence of the specific power on the load on the wing produced by the height of the UAV, etc.

V. Conclusions

The conducted analysis showed that the development of unmanned aviation systems of long-range action for flights at high altitudes provides an opportunity to significantly expand the field of application of UAVs and, in particular, dramatically reduce the estimate of satellite communications and satellite imagery. At the level of elaborating the methodology of research of HALE-class UAV, key parameters that affect the efficiency of this type of devices in accordance with a wide class of tasks were determined. A mathematical model of UAV power supply system during autonomous operation was developed, consisting of a set of solar cells, a set of electric batteries and a subsystem of transmission and distribution of electricity. The suggested mathematical apparatus is based on the following indicators: solar power function on time, the level of charge of the electric battery, performance indicators of electric batteries and performance indicators of solar cells, geographical coordinates (longitude and latitude), as well as the height of the position of the aircraft. According to the instrumentation, which included the suggested mathematical apparatus of the model of the UAV power system model, the analytical solution of the problem of optimizing the consumption and accumulation of electricity by the aircraft was obtained. The next stage of the study was to work on improving the basic model by including algorithms for the analysis of losses of electric batteries and averaging solar power function by time according to the set of time ranges that reflect the level of solar radiation power. In addition, a mathematical model of the aircraft for calculating and optimizing the UAV aerodynamic parameters was developed. This model is based on such indicators as the coefficient of lifting force, drag coefficient, air density as a function of the flight altitude and the total wing area of the aircraft. Analysis of results of modelling of UAV aerodynamic parameters of indicated the need to introduce the function of thrust of the aircraft and the coefficient of bearing capacity. As a result of the conducted research, a comprehensive method for automating the process of improving the UAV design in accordance with the task was created and methodological recommendations for UAV developers were given. The suggested approaches and developed tools for the analysis of structural features of HALE-class UAV provide an opportunity to build automated algorithms for estimating the parameters of the aircraft and to organize the procedure of expert evaluation of the design through analysis of such dependencies as the ratio of the lifting force factor on the angle of attack and the drag coefficient, the ratio of the lifting force and aerodynamic resistance as a function of the drag coefficient, the dependence of the specific power on the load on the wing produced by the height of the UAV aircraft. It should separately mentioned, that the correspondence of the suggested mathematical apparatus and the possibility of its further improvement in accordance with the specific task should be checked on the basis of experimental tests.

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