



GLOBAL JOURNAL OF RESEARCHES IN ENGINEERING: E
CIVIL AND STRUCTURAL ENGINEERING
Volume 22 Issue 1 Version 1.0 Year 2022
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4596 & Print ISSN: 0975-5861

A Structural Damage Identification Method based on Arrangement of Static Force Residual Vector

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GJRE-E Classification: *DDC Code: 572.86 LCC Code: QH450.2*



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A Structural Damage Identification Method based on Arrangement of Static Force Residual Vector

Jibao Shen ^α, Zhike Li ^σ, Shuai Luo ^ρ & Wei Wang ^ω

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Keywords: damage identification; force residual vector; static response; stiffness matrix.

1. INTRODUCTION

With the time in the service period of structure, the damage of the structure often brings potential safety hazard. Structural damage identification technology plays an increasingly important role in structural health monitoring. Reliable and effective nondestructive identification can ensure structural safety and integrity ^[1-3].

In generally, the structural damage detection is mainly to identify the location and the damage level of structural. In the process of identification, the response of the external excitation is measured by dynamic test or static test. Therefore, the damage is usually directly identified by numerical operation. The result of structural damage is the reduction of the local stiffness of the structure. Therefore, the damage of the structure can be regarded as a change in stiffness, ignoring the change in quality, and can be detected by changes in dynamic or static characteristics ^[4-8].

Considering the nature of the measurement data, the measurement methods can be divided into two categories: dynamic methods and static methods. Due to the convenience of dynamic data measurement, a

large number of damage identification methods have been developed on the basis of dynamic testing ^[9-12]. Many experts have done so much research work on damage identification using residual force vector method under dynamic response. Zimmerman et al. ^[13] firstly proposed the theoretical algorithm related to the residual force vector method. Kahl et al. ^[14] improved the theoretical algorithm proposed by the former and identified the damage location in the beam member. Li et al. ^[15] used the difference of the virtual residual force vector of the intact structure and the damaged structure to locate the damage location, combined with the response sensitivity method to identify the local damage degree, and better identified the location and damage degree of a single damage and multiple damages. Nobahariet al. ^[16] used the concept of residual force vector and proposed a method based on the damage index of truss units. This method can find the most likely damaged component location, and eliminate the undamaged units from all variables to reduce the amount of calculation, and then use the genetic algorithm to find a more specific damage unit in the concentrated position of the damaged component and calculate its damage degrees.

The purpose of dynamic analysis is to determine the parameters such as internal force, stress and displacement under dynamic load. In vibration modal analysis, the main calculation work is to solve the Eigen problem, which requires more calculation work than static analysis ^[17]. The general residual force vector method for damage identification is mainly based on the modal parameters of the dynamic test, which requires more complicated modal analysis, and the accuracy of the damage analysis results is not high. Based on the residual force vector method under dynamic response, this paper proposes a force residual vector method. This method uses the sparse property of the damage unit stiffness matrix and its corresponding residual vector distribution rule to realize the location of the damage unit and gives damage degree after the location by solving the self-balance equation of the damage unit. In this study, numerical examples of simply supported beams and trusses have been verified, and the damage location and damage degree of structure have been identified.

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II. THEORY OF FORCE RESIDUAL VECTOR METHOD

a) Basic Assumptions

In damage identification with the force residual vector method, it is necessary to make some basic assumptions about the structure:

1. When the structure is damaged, only the rigidity of the structure is reduced, and the influence of quality on the structure is ignored.
2. The damage of structural units is discontinuous in the finite element model. This assumption has been given in previous studies^[18-19].

b) Theoretical Equations

A structural system produces node displacement D under the action of static force F , the static equilibrium equation in the global coordinate system can be expressed as the following formula.

$$KD = F \quad (1)$$

where K is the global stiffness matrix of the structural system, and D is the node displacement vector in the global coordinate system.

When the structure is damaged, its stiffness matrix will change. Assuming that α_i is the damage degree of the stiffness matrix corresponding to the i -th structural unit, the perturbation matrix ΔK of structural damage can be expressed as the following formula:

$$\Delta K = \alpha K \quad (2)$$

where α is the damage degree vector.

Substituting the damage stiffness ΔK and the displacement d of the structure after damage into Eq. (1), we can get the following formula:

$$(K - \Delta K)d = F \quad (3)$$

For Eq. (3), moving the term without ΔK to the right side of the equation, we can get the following equation.

$$\Delta Kd = Kd - F \quad (4)$$

Define the vector on the right side of Eq. (4) as the force residual vector P .

$$Kd - F = P = [p_1 \quad p_2 \quad \cdots \quad p_n]^T \quad (5)$$

The values of elements p_i corresponding to the damaged units in the P vector are much larger than those of the undamaged units. The values of elements p_i corresponding to the undamaged units approach 0. That can be used as a filtering condition in the analysis process. The elements in the vector P of Eq. (5) are arranged in descending order of their absolute values. The structure units with larger values in front are

corresponding to the damaged units. It can be used to realize localization of damage unit precisely.

Eq. (5) is composed of n equilibrium equations at nodes, Suppose that there is an unit damage on a node, and that the elements p_i values, the unit stiffness matrix and the displacement vector are proportional for a given damage unit, the ratio is the damage degree. It can be realized identification of damage degree. Therefore, the damage coefficient can be obtained according to the following formula:

$$\alpha_i K_i d = P_i \quad (6)$$

K_i is the global stiffness matrix containing only the unit stiffness matrix elements of the damage unit.

c) Solving Steps

The specific solving steps of this method are as follows:

1. Establish a stiffness matrix for the target structure and combine the static balance equation to obtain the force residual vector P .
2. List the elements of vector P according to their corresponding degrees of freedom of structure units, and arrange them in descending order of absolute value.
3. For the arranged absolute value sequence, divide the previous element by the next element. The position where the quotient obtained by the calculation tends to infinity is the last damaged unit, and the value of degree of freedom about the last damaged unit is expressed as the total amount of degree of freedom about the damaged units. According to the fact that a structure unit has four degrees of freedom, the number of damage units can be judged.
4. For the elements in the list arranged according to the structure unit number, the structure unit whose ratio of the vector value corresponding to the first displacement and the third displacement equals -1 is the damage unit, so that the damage unit is located. And verify the judgment result of step (3).
5. Take out the structure unit stiffness matrix of the located damage unit in the global coordinate system, establish the node balance equation one by one according to the number of the damage unit, and solve the damage degree of each unit according to equation (6) to determine the damage level of the structural member.

Fig.1 shows the flow chart of the specific solving steps of this method.

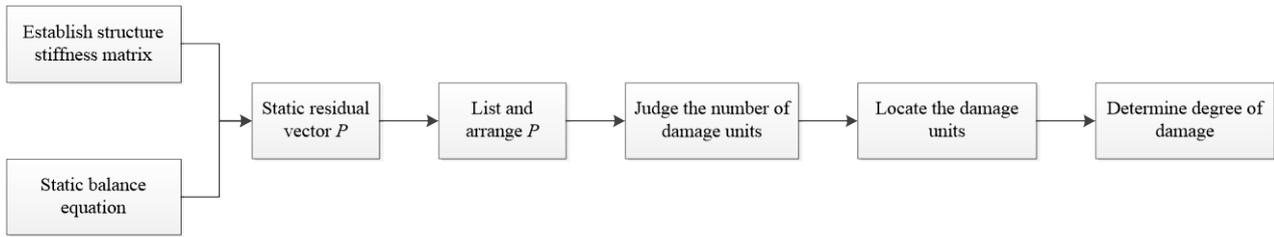


Fig. 1: Flow chart of calculation

III. NUMERICAL MODEL EXAMPLES

In order to apply the force residual vector to practice, the following takes simply supported beams and truss structures as examples for numerical model calculations. At first we set the degrees of damage of some structural units, and calculate the displacements of the structural unit nodes by calculating the force residual vector. Then the node displacements are used to locate the damage units and to determine the degrees of damage. If the identified location and the identified degrees of damage are consistent with the set values, it can show that the force residual vector method can be used to identify the damage location and the damage degree of the structure through the values of node displacements.

a) Numerical Model Example of Simply Supported Beam

i. Calculated Displacement Value of Simply Supported Beam

Taking the simply supported beam model shown in Fig.2 as an example to explain the proposed method. The simply supported beam structure is divided into 12 units, and then the finite element analysis for it is carried out. The basic parameters of No.14I-beam are as follows: unit length $L=0.1\text{m}$, cross-sectional area $A=2.15 \times 10^{-3}\text{m}^2$, elastic modulus $E=200\text{GPa}$, moment of inertia $I=7.12 \times 10^{-6}\text{m}^4$, density $\rho=7.8 \times 10^3\text{kg/m}^3$. The simply supported beam model structure is added a downward force $F=10\text{kN}$ in the middle of the span, constrained the horizontal and vertical displacement at node 1, and constrained the vertical displacement at node 13.

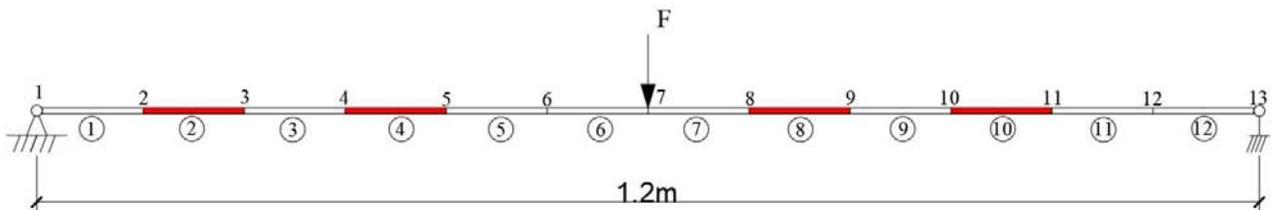


Fig. 2: Sketch of simply supported beam

Regardless of the axial displacement of the simply supported beam model, the stiffness matrix of the beam units related to the vertical and angular displacement is taken as^[13-14]:

$$K_c = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \quad (7)$$

According to the degrees of damage α_i ($0 < \alpha_i < 1$) introduced Eq.(2), the global stiffness matrix of the beam can be expressed as:

$$K = \sum_{i=1}^{12} (1 - \alpha_i) K_i \quad (8)$$

where K_i represents the i -th unit stiffness matrix in the global coordinate system, and the subscript i represents the unit number.

Table 1 shows the node numbers and the numbers of the node degrees of freedom of simply supported beam units.

Table 1: Node number and number of node degrees of freedom of units

Node number		1	2	3	4	5	6	7	8	9	10	11	12	13
Number of node degrees of freedom	Vertical displacement	—	2	4	6	8	10	12	14	16	18	20	22	—
	Angular displacement	1	3	5	7	9	11	13	15	17	19	21	23	24

Take multiple damage case as an example: the loss of stiffness is 15% for unit 2, 60% for unit 4, 13% for unit 8, and 30% for unit 10, that is $\alpha_2=0.15$, $\alpha_4=0.60$, $\alpha_8=0.13$ and $\alpha_{10}=0.30$.

The node displacements are calculated according to Eq.(3), and arranged in Table 2 according

to the order of degrees of freedom of the units. In the table, v_a and θ_a are the vertical and rotational displacements of the left node of an unit, and v_b and θ_b the vertical and rotational displacements of the right node of an unit.

Table 2: Node displacement of simply supported beam (mm)

Displacement	Unit					
	1	2	3	4	5	6
v_a	0.000	-0.787	-0.770	-0.708	-0.620	-0.313
θ_a	0.000	-0.078	-0.152	-0.219	-0.266	-0.290
v_b	-0.787	-0.770	-0.708	-0.620	-0.313	-0.155
θ_b	-0.078	-0.152	-0.219	-0.266	-0.290	-0.296

Displacement	Unit					
	7	8	9	10	11	12
v_a	-0.155	0.039	0.232	0.413	0.536	0.662
θ_a	-0.296	-0.282	-0.250	-0.202	-0.142	-0.073
v_b	0.039	0.232	0.413	0.536	0.662	0.714
θ_b	-0.282	-0.250	-0.202	-0.142	-0.073	0.732

Fig. 3 shows the deformation diagram of the simply supported beam. In the figure, the dotted lines

present the original structure, and the solid lines present the deformed structure.

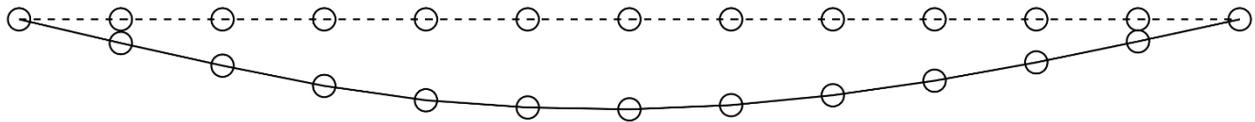


Fig. 3: Deformation diagram of simply supported beam

ii. Damage Identification of Simply Supported Beam

The calculated node displacements in Table 2 are used as the known values of structural damage identification.

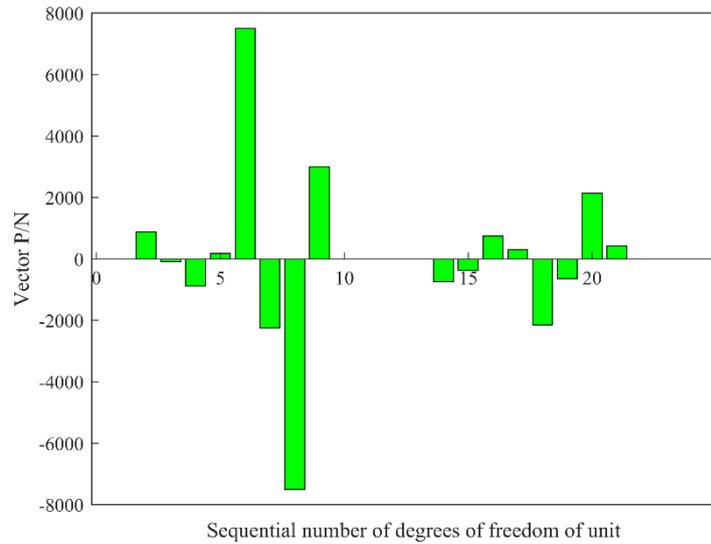
The force residual vector P is determined according to Eq.(5) and shown in Table 3.

Table 3: Values of force residual vector P (N)

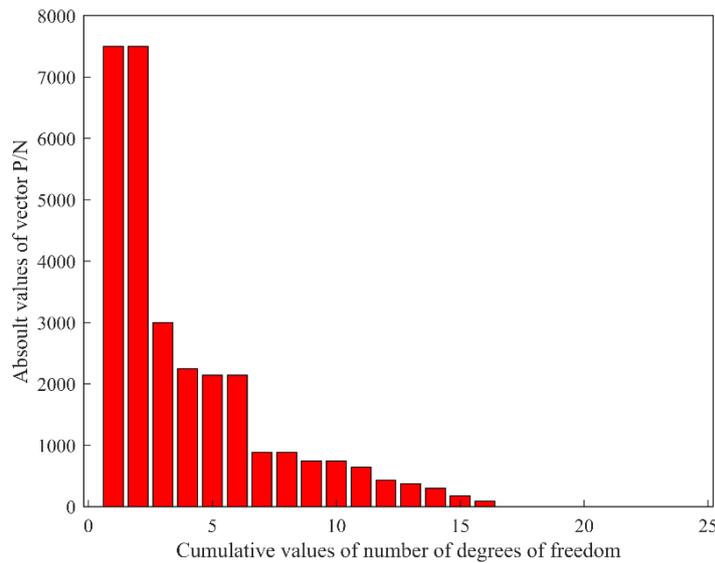
Displacement	Unit					
	1	2	3	4	5	6
v_a	0	882.535	-888.35	7500	-7500	0
θ_a	0	-88.24	176.47	-2250	3000	0
v_b	882.535	-888.35	7500	-7500	0	0
θ_b	-88.24	176.47	-2250	3000	0	0

Displacement	Unit					
	7	8	9	10	11	12
v_a	0	-747.13	747.13	-2142.90	2142.9	0
θ_a	0	-373.56	298.85	-642.86	428.57	0
v_b	-747.13	747.13	-2142.90	2142.9	0	0
θ_b	-373.56	298.85	-642.86	428.57	0	0

Fig. 4 shows the distribution and arrangement of force residual vector P .



(a) Distribution diagram of residual vector value



(b) Arrangement diagram of absolute value of residual vector

Fig. 4: Distribution and arrangement of residual vector values of simply supported beam

Fig. 4 (a) presents the distribution of force residual vector P . The abscissa represents from left to right, the sequential number of vertical and rotational degrees of freedom of unit nodes. The ordinate corresponds to the force residual vector of corresponding units. Fig.4 (b) shows the arrangement of the absolute values of P in descending order. The abscissa represents the cumulative values of the number of vertical and rotational degrees of freedom of unit left and right nodes.

For the arranged absolute value sequence in Fig.4 (b), divide the previous element by the next element, we obtain the positioning diagram shown in

Fig.5. The maximum ratio which represents the total number of degrees of freedom of the damage units is located at 16. Since each unit corresponds to 4 degrees of freedom in the local coordinate system, it can be seen that four units in the simply supported beam have been damaged, which is consistent with the set number of damage units.

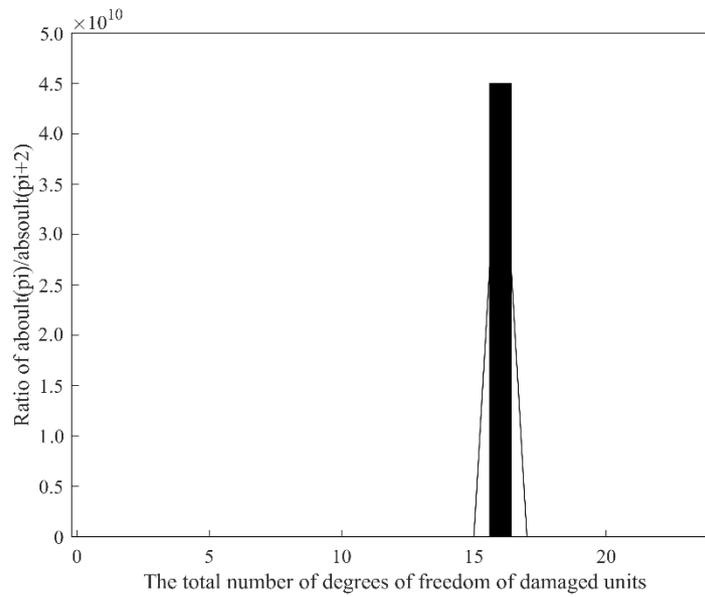


Fig. 5: Total number of freedom degrees of damage elements

In Table 3, the units 2, 4, 8 and 10 whose ratio of the residual vector values P corresponding to the first displacement and the third displacement of the units is -1 are damage units. Those located damage units are the same as the set damage units.

The identification values of damage degree obtained by solving damage degrees using Eq.(6), and the setting values of the corresponding units are listed in Table 4.

Table 4: Identification values and setting values of unit damage degrees of simply supported beam

Damage unit number	2	4	8	10
Setting value	0.15	0.60	0.13	0.3
Identification values	0.15	0.60	0.13	0.3

It can be seen from Table 4 that all the identification values of damage degrees are completely consistent with the corresponding setting values. It can be concluded that the force residual vector method can be used to locate the damage position and identify the damage degree of simply supported beam structure.

elements which are all bar units, the length of the bottom and upper horizontal bar units is 0.4m, 0.3m for the vertical bar units, and 0.5m for the diagonal bar units. The elastic modulus of the steel used is $E=200\text{GPa}$, the cross-sectional area of the L-shaped steel unit is $A=2.276 \times 10^{-4}\text{m}^2$, the density $\rho=7.8 \times 10^3\text{kg/m}^3$, and the vertical concentrated force $F=15\text{kN}$ is loaded at the bottom in the middle of the span.

b) Example of Truss Numerical Model

i. Calculated Value of Truss Displacement

The specific numerical model size of the truss structure is shown in Fig. 6. The truss has 10 spans, 37

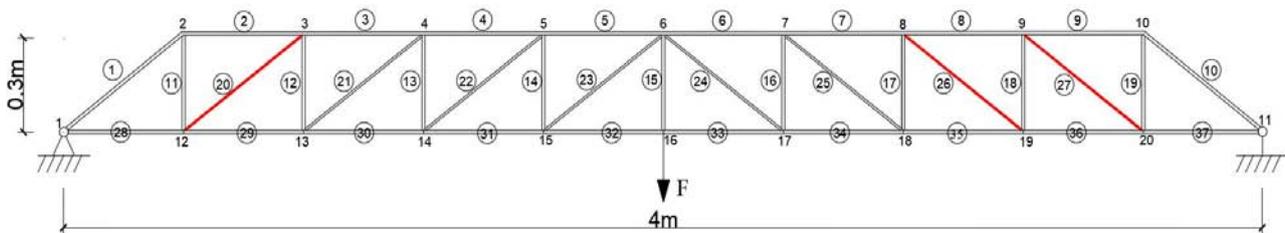


Fig. 6: Sketch of truss model

The unit stiffness matrix in the local coordinate system of the unit is expressed as

$$K_e = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (9)$$

where E is the elastic modulus of the unit, A is the cross-sectional area of the unit, and L is the length of the unit.

The transformation matrix of unit stiffness matrix in local coordinate system into that in global coordinate system can be expressed as

$$K_g^e = \frac{EA}{L} \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta & -\cos^2 \theta & -\cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta & -\cos \theta \sin \theta & -\sin^2 \theta \\ -\cos^2 \theta & -\cos \theta \sin \theta & \cos^2 \theta & \cos \theta \sin \theta \\ -\cos \theta \sin \theta & -\sin^2 \theta & \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix} \quad (11)$$

In this example, the structure has 20 nodes with 40 degrees of freedom. The association table method^[15-16] requires an extraction matrix $T_{4 \times 40}$. The matrix T extracts the degrees of freedom of different units in the global coordinate system. The T matrix changes with the degrees of freedom of unit and is expressed as

$$T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}_{4 \times 40} \quad (12)$$

The unit stiffness matrix in the global coordinate system is

$$K_g^E = T^T K_g^e T \quad (13)$$

where K_g^E is 40×40 order matrix.

According to Eq. (2) the damage coefficient α_i ($0 < \alpha_i < 1$) is introduced, and the global stiffness matrix of the truss can be expressed as

$$K = \sum_{i=1}^{37} (1 - \alpha_i) K_i \quad (14)$$

where K_i represents the i -th unit stiffness matrix in the global coordinate system, and the subscript i represents the unit number.

In order to obtain the static response of the structure, the vertical concentrated force $F = 15\text{KN}$ is loaded at the bottom in the middle of the span. The truss model is modeled by the correlation table method^[15-16]. The steps of locating the position of the stiffness matrix by the correlation table method are as follows: assume that the node (i, j) of the unit N corresponds to the position of the node degrees of freedom of the global stiffness matrix as $N_i(2 \times i - 1, 2 \times i)$,

$$S = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ 0 & 0 & \cos \theta & \sin \theta \end{bmatrix} \quad (10)$$

θ is expressed as the rotation angle of the unit between the local coordinate system and the global coordinate system.

The given unit stiffness matrix and transformation matrix under the local coordinate system of the unit, the unit stiffness matrix under the local coordinate system can be transformed into the unit stiffness matrix under the global coordinate system, and the unit stiffness matrix K_g^e (4×4 order matrix) under the global coordinate system can be obtained.

$N_j(2 \times j - 1, 2 \times j)$. For example, for unit 12, the node numbers are 3 and 13, and the position of the node degrees of freedom corresponds to the global stiffness matrix are 5, 6, 25 and 26. Therefore, the corresponding positions of unit 12 in the global stiffness matrix are 5 rows, 6 rows, 25 rows, 26 rows, 5 columns, 6 columns, 25 columns and 26 columns.

In order to better simulate the actual damage situation, different damage coefficient values are set for different units of the truss. Assume that Units 20, 26 and 27 are damaged by 15%, 60%, and 25%, that is, $\alpha_{20}=0.15$, $\alpha_{26}=0.60$ and $\alpha_{27}=0.25$. Use Eq.(3) to solve the displacement values of each node after damage, as shown in Table 5. In the table, u and v represent horizontal and vertical displacements respectively.

Table 5: Displacement values of unit nodes (mm)

Displacement	Node									
	1	2	3	4	5	6	7	8	9	10
u	0.000	2.225	2.137	1.962	1.698	1.347	0.995	0.731	0.556	0.468
v	0.000	-3.196	-6.247	-8.790	-10.629	-11.531	-10.705	-8.941	-6.131	-3.120

Displacement	Node									
	11	12	13	14	15	16	17	18	19	20
u	2.636	0.088	0.264	0.527	0.879	1.318	1.757	2.109	2.373	2.548
v	0.000	-3.245	-6.297	-8.839	-10.678	-11.630	-10.754	-8.991	-6.181	-3.169

Fig. 7 shows the deformation diagram of the truss. In the figure, the dotted lines present the original structure, and the solid lines present the deformed structure.

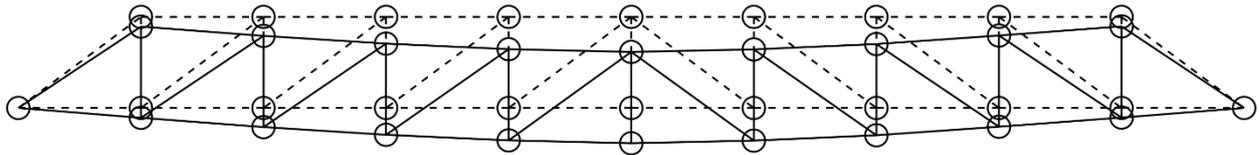


Fig. 7: Deformation diagram of truss

ii. Truss Damage Identification

Use Table 5 to calculate the node displacement value as the known value for structural damage identification.

According to Eq.(5), the force residual vector P is obtained, which is listed in Table 6. In the table, u_a

and v_a are the horizontal and vertical vectors of the premier node of an unit respectively, and u_b and v_b the horizontal and vertical vectors of the second node of the unit.

Table 6: Values of force residual vector P (N)

Displacement	Unit							
	1	2	3	4	5	6	7	8
u_a	0	0	-1765	0	0	0	0	15000
v_a	0	0	-1324	0	0	0	0	-11250
u_b	0	-1765	0	0	0	0	15000	3333
v_b	0	-1324	0	0	0	0	-11250	-2500

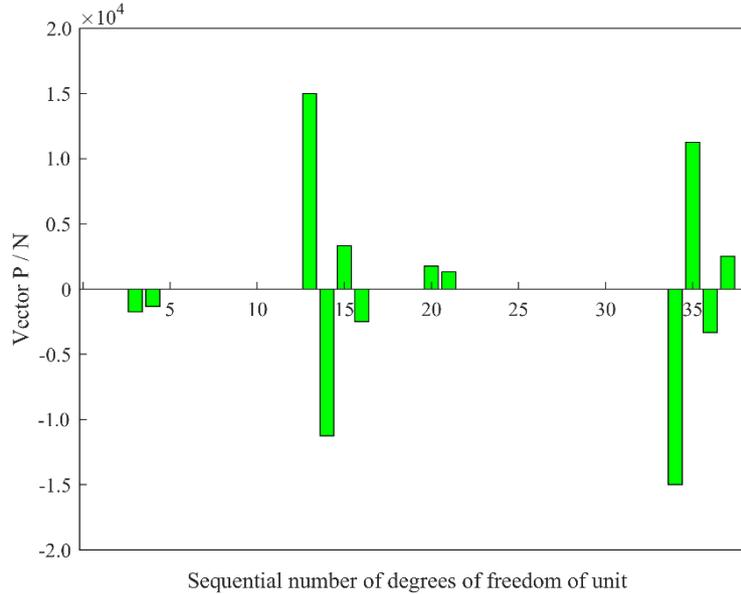
Displacement	Unit							
	9	10	11	12	13	14	15	16
u_a	3333	0	0	-1765	0	0	0	0
v_a	-2500	0	0	-1324	0	0	0	0
u_b	0	0	1765	0	0	0	0	0
v_b	0	0	1324	0	0	0	0	0

Displacement	Unit							
	17	18	19	20	21	22	23	24
u_a	15000	3333	0	-1765	0	0	0	0
v_a	-11250	-2500	0	-1324	0	0	0	0
u_b	0	-15000	-3333	1765	0	0	0	0
v_b	0	11250	2500	1324	0	0	0	0

Displacement	Unit							
	25	26	27	28	29	30	31	32
u_a	0	15000	3333	0	1765	0	0	0
v_a	0	-11250	-2500	0	1324	0	0	0
u_b	0	-15000	-3333	1765	0	0	0	0
v_b	0	11250	2500	1324	0	0	0	0

Displacement	Unit				
	33	34	35	36	37
u_a	0	0	0	-15000	-3333
v_a	0	0	0	11250	2500
u_b	0	0	-15000	-3333	0
v_b	0	0	11250	2500	0

Fig. 8 represents a distribution and arrangement diagram of the force residual vector P .



For the arranged absolute value sequence in Fig. 8 (b), divide the previous element by the next element, we obtain the positioning diagram shown in Fig. 9. The maximum ratio which represents the total number of degrees of freedom of the damage units is

located at 12. Since each unit corresponds to 4 degrees of freedom in the local coordinate system, it can be seen that three units in the truss have been damaged, which is consistent with the set number of damage units.

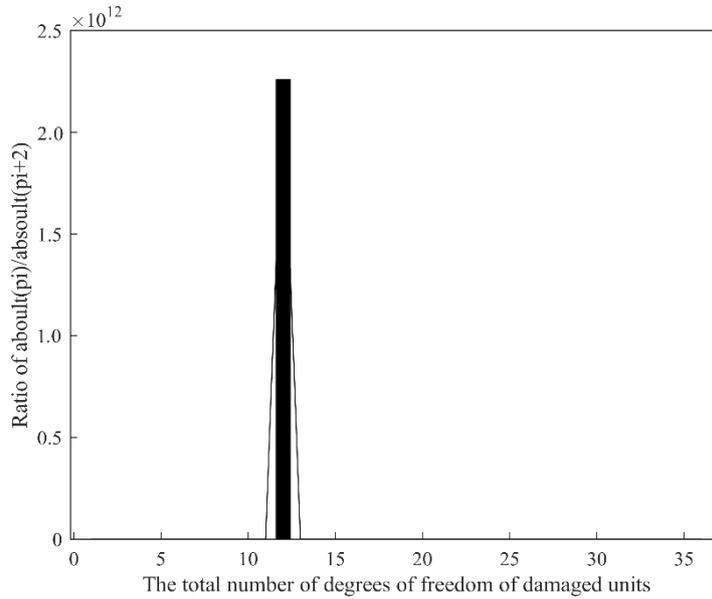


Fig. 9: Total number of freedom degrees of damage units

In Table 6, the truss units 20, 26 and 27 whose ratios of the residual stress vector value P corresponding to the first displacement and the third displacement of the element are -1 is the damaged units, and the damaged units located are the same as the set damaged units.

The identification values of damage degree obtained by solving damage degrees using Eq.(6), and the setting values of the corresponding units are listed in Table 7.

Table 7: Identification values and setting values of unit damage degrees of truss

Damage unit number	20	26	27
Setting value	0.15	0.60	0.25
Identification values	0.15	0.60	0.25

It can be seen from Table 7 that all the identification values of damage degrees are completely consistent with the corresponding setting values. It can be concluded that the force residual vector method can be used to locate the damage position and identify the damage degree of truss structure.

Eraky et al. [20] used the dynamic test method to obtain the eigenvalues and eigenvectors of the structure, and combined with the residual force vector method to identify the damage of the structure. In this paper, the static displacement parameters are obtained by the static test method. The static displacement

parameters are easier to obtain than the dynamic parameters, and the accuracy of the results is more accurate. In addition, this paper uses the intelligent force residual vector algorithm to obtain more accurate results and faster damage identification.

IV. CONCLUSION

Based on the existing research results of residual force vector method, a new structural damage identification method based on force residual vector is proposed in this paper. Through the identification analysis of some units of the simply supported beam numerical model and the truss numerical model under different damage degrees at the same time, It is shown that this method uses the arrangement of force residual vector elements to intelligently obtain the location of damage recognition, the number of damage elements and the degree of damage.

It only needs the displacement information under static load and does not need complex modal analysis. In addition, because of the addition of substructure, the solution of the system will not be complicated, and the damage identification result is faster and the calculation speed has a great advantage.

Conflicts of Interest

The authors declares that there are no conflicts of interest.

Data Availability

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

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