Artificial Intelligence formulated this projection for compatibility purposes from the original article published at Global Journals. However, this technology is currently in beta. *Therefore, kindly ignore odd layouts, missed formulae, text, tables, or figures.*

New Full Wave Theory for Plane Wave Scattering by a Rough Dielectric Suface - The Correction Current Method

Henry Tsai

Received: 7 December 2020 Accepted: 1 January 2021 Published: 15 January 2021

6 Abstract

3

4

A new full wave theory for scattering by one dimensional perfectly conducting rough surface 7 has been formulated recently. It provides enhanced physical insights into rough surface scattering processes, includes multiple scattering effects, quantifies field errors and furnishes a 9 quantitative measure of the method?s accuracy, permits a systematic procedure for obtaining 10 higher-order terms in the iterative solution of the scatter problem, and satisfies reciprocity 11 using only the first-order solution. The first- order solution of this new full wave method has 12 been shown to reduce to the small perturbation and the Kirchhoff approximation in their 13 regions of validity. It has also been numerically applied to surfaces with Gaussian height and 14 slope variations and shown to be more accurate than the small-perturbation and the Kirchhoff 15 methods in regions where neither are considered valid. This paper extend the theory to the 16 more general and important case of scattering by a dielectric interface, where one of the two 17 halfspaces is lossy 18

19

Index terms— Felix Schwering?, Gerald Whitman? & Henry Tsai? Abstract-A new full wave theory for scattering by 20 21 one dimensional perfectly conducting rough surface has been formulated recently. It provides enhanced physical 22 insights into rough surface scattering processes, includes multiple scattering effects, quantifies field errors and 23 furnishes a quantitative measure of the method's accuracy, permits a systematic procedure for obtaining higher-24 order terms in the iterative solution of the scatter problem, and satisfies reciprocity using only the first-order 25 solution. The first-order solution of this new full wave method has been shown to reduce to the small perturbation 26 and the Kirchhoff approximation in their regions of validity. It has also been numerically applied to surfaces 27 with Gaussian height and slope variations and shown to be more accurate than the small-perturbation and the 28 Kirchhoff methods in regions where neither are considered valid. This paper extend the theory to the more 29 general and important case of scattering by a dielectric interface, where one of the two halfspaces is lossy. 30

31 **1 I**.

Formulation of Problem by use of CC-Method ubject of this paper is a new theory of scattering from rough dielectric surfaces of the type shown in Fig 1. The interface between an air halfspace and a dielectric halfspace is rough over a length 2L and planar beyond this region. The roughness profile is onedimensional, i.e. the local height D varies with z but is constant with y.

 $_{39}$ The technique used in this paper to formulate the problem is the Correction Current (CC) method, a new

full wave method which has recently been established for plane wave scattering from metal surfaces [1] and subsequently extended to dielectric surfaces, with the lower halfspace at first assumed to be lossless but in the much presented here allowed to be lossless 1

42 work presented here allowed to be lossy 1.

43 Author ? ?: US Army CERDEC Fort Monmouth, NJ. Author ?: NJIT Newark, NJ. e-mail: ger-44 ald.whitman@njit.edu The CC-method defines a primary field and a complete system of scatter fields, called

A plane wave is incident under the angle ? 0 upon the interface; it may be incident from above or below. This incident wave and the resulting scattering field are TE-polarized, i.e. these fields consist of components E y , H x and H z .

radiation modes. Each of these fields consists of an incident, reflected and transmitted plane wave. These 45 component waves, however, are modified such that each of the radiation modes as well as the primary field 46 satisfies the boundary conditions at the interface, individually and rigorously. Since these fields are simple in 47 48 structure (i.e. consisting of suitably modified plane waves) and comply with the boundary conditions at the rough interface, they will no longer satisfy Maxwell's equations 1. This is remedied by introducing fictitious current 49 distributions, called passive currents that are assumed to be associated with each of these fields and chosen such 50 that they exactly cancel the field errors of these fields; the passive current distribution of course varies from mode 51 to mode. Thus, while not being solutions of Maxwell's source free equations, the primary field and radiation 52 modes are solutions of Maxwell's equations with sources 2. The passive currents exist only in the corrugation 53 region |Z| ? L; outside this region, where the interface is planar, the primary field and radiation modes are exact 54 solutions of Maxwell's source free equations and no correction is needed. 55

⁵⁶ 2 S Air Halfspace

57 ? z O

58 3 2L Dielectric Halfspace

In addition, it is assumed that each radiation mode is generated by an "active" sheet current distribution residing 59 in a plane z = termed the reference plane of this mode. These reference planes exist in the corrugation region |Z|60 ? L only, and all active currents of the mode system are limited to the same region as the passive currents. An 61 important feature of these active currents is that taken together they form a complete orthogonal system so that 62 63 any (passive) current distribution in the region |Z|? L can be expanded into, or nullified by the active currents. 64 The total field is then written as a superposition of the primary field and the radiation modes. This combined 65 field, of course, is not allowed to contain any active or passive current distribution, and this "zerocurrent" condition is then used to determine the unknown amplitudes of the radiation modes by an iterative procedure 66 which in, a step-by-step fashion, eliminates the passive currents of the primary field and the radiation modes by 67 the active currents of the radiation modes. The completeness, mentioned above, of the active currents allows us 68 to do this consistently. The resulting combined field will satisfy the boundary conditions at the interface and 69 the radiation condition at infinity while all active and passive currents are eliminated by mutual compensation. 70 Thus the combined field is the solution of the scatter problem. For numerical efficiency, the iterative process is 71 72 typically cut-off after the first iteration. Using this approach the dielectric surface scatter problem has recently been solved for the case that the 73

Using this approach the dielectric surface scatter problem has recently been solved for the case that the dielectric halfspace is loss free. Scattering patterns have been obtained in the form of single integrals over elementary functions which are easily evaluated numerically. The patterns have been computed for both deterministic and random rough surfaces. Comparison with the corresponding patterns obtained by a Method of Moments (MoM) procedure has shown that the first order iteration theory is of good accuracy over a wide parameter region. A paper reporting on this study, which was conducted at CERDEC and NJIT, is close to completion and will be submitted for publication in the near future.

The extension of the theory to lossy dielectrics is currently pursued under the 2007 In-house Laboratory 80 Independent Research (ILIR) program. This extension is not straight forward, however. The problem geometry 81 consists of two halfspaces; and two groups of radiation modes are required for full characterization of the problem. 82 This holds for the lossy as well as for the lossless case. But when these two mode groups for the lossy case are 83 defined in direct analogy to the two mode groups for the lossless case one of the groups diverges, i.e. increases 84 exponentially with $|\mathbf{x}|$, which is unacceptable. Hence the radiation modes need to be redefined, which has a 85 significant effect on the theory. In the development of this theory it is consistently assumed that [? r] is finite; 86 it may be small, but is not allowed to be arbitrarily small. Comparison to the results of the lossless theory will 87 be made only after all formulas for fields and radiation patterns have been fully established. 88

⁸⁹ 4 II. Analytical Results and Discussion

In the following it is assumed that the incident plane wave is situated in the upper (air) halfspace. If it is located
in the lower halfspace, the expressions obtained for fields and patterns will be different, of course. But the overall
trends observed are similar.

It is interesting to note that the scatter fields associated with the two locations of the incident wave are related by a simple symmetry relation. Assume that a plane wave of amplitude ?? 0 and phase constant (in the z-direction) incident in the air halfspace -generates a scatter field ? 0 -?? s (x,z;k 0, k ?, D(z)). Assume, furthermore, that a plane wave of amplitude E ? and phase constant ? ? -incident in the dielectric halfspace

⁹⁷ -creates a scatter field ?? ? y s (x,z;k 0 ,k ? , D(z)). Then these two scatter fields are related by the condition. ⁹⁸ for? ? = ? 0 with k ? = k 0 ?? ??

⁹⁹ This relation holds for all x and z, i.e. in both halfspaces. Thus, if ?? y s has been determined ?? ? y s , ¹⁰⁰ follows by a simple substitution of coordinates and parameters 3 ; and vice versa.

Symmetry relation (1) must be satisfied for the exact scatter fields. But it is satisfied already for the first order iteration approximation used in this paper which may be taken as an indication that the first order method is of good accuracy.

2

In this first order approximation the CC-theory leads to the following expression for the scatter field in the 104 upper (air) halfspace x > D:()()) 1 () , ; ; () , ; ; (0 0 0E z D k k z x E E z D k k z x E s y s y ? ? = 105 ? ? ? D x for a z d du u e e e D u v v u v u z z j u u e z d du u e z z j z j D v u j x jv D u u j u L L z D u u j z 106 107 $+?+?+?=+?=?+??????)2(11)(2112\sim22\sim)()(2020202000)(1\sim20202000))$ 108 109 110 = + ? = + + ? ?)()(-11)(211)(,1)2(1)(2100220100)(0000)1(00???)111

Where E 0 is the amplitude of the incident plane wave ??? stands for D(??) and k 0 is the free-space wave number; furthermore

The path of the u-integration runs along the positive real axis 4. The propagation constants v 0, v 1, ? 1 and 2 are positive real and/or negative imaginary while is positive real and positive imaginary.

The first double integral in (2a) represents the contribution of the first mode group to the scatter field and 116 the second double integral shows the contribution of the second mode group. A formally similar representation is 117 obtained for the scatter field in the lower (lossy dielectric) halfspace but for brevity is not spelled out here. In the 118 far field region, where eq. (2) can be simplified by substituting where the scatter angle ?? is in the region The 119 120 formula may then be evaluated asymptotically for k 0??? using the method of stationary phase which eliminates 121 the u-integration. Only the first double integral in (2a) contributes to the far field and one obtains where the scatter pattern S 0 (??, ?? 0) takes the form []()[]() 2 1 2 2 0 2 2 1 2 0 2 2 1 2 0 1 2 1 2 0 2 1 0 0 0 2 1 122 123 ???????????(2b)()??20212200»>>+ = and L k z x k k,?????sin, $\cos = z x 22???$ 124 125),()(22 126

127 **5** I

This expression for the scatter pattern in the air halfspace x>D is identical to the one obtained by the previously developed theory for ? r =real, but it is now confirmed to be valid for complex ? r as well. Eq. (3b) also shows that S 0 vanishes, as required, in the nocontrast case that ? r ?1, and that it satisfies the reciprocity relation S 0 (??, ?? 0) = S 0 (-?? 0, -??).

Similar to expression (2) for the scatter field in the upper halfspace, the formula obtained for the lower halfspace (not shown here) consists of two double integrals representing the contributions of the two mode groups. However, due to the lossy nature of the dielectric halfspace, the scatter field will decrease here exponentially away from the corrugated interface, and the two double integrals in general cannot be simplified. Simplification is possible only if Im[? r] is either very small, so that a far field of reasonable magnitude exists, or if Im[? r] is rather large, so that the scatter field in the lower halfspace is in effect confined to a neighborhood of the scatter surface.

The scatter field in this case consists of two parts which may be interpreted in the following way. In contrast 142 to the scatter field in the upper halfspace, the scatter pattern in the lower halfspace is generated by two different 143 mechanisms which are illustrated by Fig. 2. The first mechanism, as in the case of the lossless upper halfspace, is 144 simply the scattering of the incident plane wave at the corrugated part of the interface. The second mechanism 145 146 comes about because the scatter field in the lower, lossy halfspace decreases with ? exponentially, i.e. much 147 faster than the scatter field in the upper halfspace. But the scatter field must be continuous at the interface. As a consequence there is a continuous leakage of energy from the upper halfspace into the lower halfspace, which 148 constitutes the second mechanism generating the scatter field in the lower halfspace ?? . The first mechanism 149 causes a conventional scatter field where in the far zone the ?? and ??? dependence are separated, but with 150 the ?? dependence in this case decreasing exponentially; see eq.(4a). This field will dominate at scatter angle 151 ?? close to ± 180 o. In the asymptotic field caused by the second mechanism, on the other hand, the ?? and 152 ?? ? dependence remain coupled also in the asymptotic region. This part of the field will be dominant near the 153 interface, i.e. for ?? near ± 90 o. and k k where (1) Scattering of primary field at corrugated section of interface 154 (2) Energy transfer from scatter field in upper halfspace to asymptotic scatter field in lower half space A possible 155 problem is the following. If |Im[? r]| is very small then the denominator of F? in (4b) will have a near-zero at 156 157 $\sin ??=|? r|-1/2$ resulting in a sharp peak of F?. The authors are not sure that this is correct and eq. (4b) 158 will require further study.a T F S k e E E r k j y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 000)(0)1()4()(), () 159 = ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 0] cos) 1 ([sin) 4 (0 Im 0 0 Im) 1 (cos sin) , (160

As mentioned above, if Im[? r] is sizeable then eq. (??), even if mathematically correct, will lose its physical meaning. The equation holds in the far zone, i.e. at large distances from the corrugated part of the interface, and if Im[? r] is significant then the scatter field at such distances will be exceedingly small and undetectable for all practical purpose.

165 Conceptually it is obvious that, in the case of large Im[? r], the scatter field in the lower halfspace will be of

significant magnitude only in a narrow region adjacent to the interface, while this field, whether generated by the
first or the second mechanism mentioned above, will decrease rapidly with increasing distance from the interface.
The general representation of the scatter fields in terms of two double integrals (i.e. a representation akin to eq.
(2a)) allows to quantify this, but the analytical procedure is rather lengthy and tedious, and not included in this

170 paper.

One last remark: The scatter pattern (3b) in the upper halfspace includes a factor cos ?? and will be zero for 171 ± 90 o, i.e. at the interface, indicating that E y along the) r k k z x ?????? 0, sin, cos = = = () r k k z 172 x???????0, sin, cos = = 210000, 2 z k x k and L k z k <>>>>? interface declines faster than 173 The scatter field in the direct vicinity of the interface can be determined by an asymptotic evaluation of (2) 174 for i.e. in a region where z is large but x remains small compared to z (Fresnel Region). One obtains The formula 175 shows that E y near the interface decrease with , i.e. the energy density will decrease here with ? -3 rather than 176 with ? -1 , the rate of decrease for | ?? |<90 o . With x, the field below the surface decreases exponentially, 177 indicating a transmitted wave while above the surface it varies linearly with x indicating the interaction between 178 an incident and a reflected wave. All this is consistent with a continuous leakage of energy from the air halfspace 179 into the lossy dielectric halfspace, i.e. with the second mechanism mentioned above for generating the scatter 180 field in the lower halfspace.2 1 0 0 z k e z jk ? ? + ? = + ? ? ? ? ? ? ? + ? = L L z D ju y u e v u u k E j E ~0 181 182) (,) ()], () 1 (1 [) 1 (**1** 1 183

As mentioned earlier, the discussion and formulas presented in this paper apply to the case that the primary wave is incident in the upper (air) halfspace. If the antenna generating the incident wave is situated in the lower (dielectric) halfspace, the symmetry relation (1) applies; the formulas are analogous; and similar overall trends are observed.

The theory for the lossy dielectric case summarized in this paper has been developed under the 2007 CERDEC ILIR program in cooperation with NJIT and is near completion, though some points need further investigation. Final results will be tested by comparison to data obtained by a MoM technique. This work is schedule for 2008. Numerical Techniques such as the MoM and FDTD methods can be relied on to provide very accurate results. Analytical methods as the one presented in this paper, even though approximate, have the advantage of showing parameter dependencies explicitly, thus providing physical insight. In addition, the field and pattern formulas

-often obtained in the form of single integrals over elementary functions -are amenable to efficient computer evaluation and may be useful for real-time modeling.

1

Figure 1: Fig. 1 :

Figure 2: Fig. 2 :

Figure 3:

195

¹For a review of established theoretical methods for the analysis of rough surface scattering see[2]-[4].2 Alternatively, the passive currents may be interpreted as quantifying the field errors.

²Note that since ?? = (? ? /?0) 2 the parameter substitution replaces ?r by 1/? r.

 $^{^{3}}$ Except when a (u-u') -1 singularity occurs on the real u-axis. In this case the u-integration bypasses the singularity by a local deformation of the integration path into the positive imaginary u-halfspace.

 $^{^4\}mathrm{No}$ such energy transfer occurs when ? r is real, since the energy density in both half spaces decreases with ? -1 .

- 196 [Schwering et al. ()] 'New full-wave theory for scattering from rough metal surfaces-the correction current
- method: the TE-polarization case'. F K Schwering , G M Whitman , A A Triolo . Waves and Random
 Media, 2004. 14 p. .
- [Beckmann and Spizzichino ()] The Scattering of Electromagnetic Wave from Rough Surfaces, P Beckmann , A
 Spizzichino . 1963. New York: Macmillan.
- [Ogilvy ()] Theory of Wave Scattering from Random Rough Surfaces, J Ogilvy . 1991. Philadelphia Institute of
 Physics
- 203 [Voronovich ()] Wave Scattering from Rough Surfaces, A G Voronovich . 1998. Berlin: Springer Verlag. (2nd ed.)