

New Full Wave Theory for Plane Wave Scattering by a Rough Dielectric Surface -The Correction Current Method

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Abstract

A new full wave theory for scattering by one dimensional perfectly conducting rough surface has been formulated recently. It provides enhanced physical insights into rough surface scattering processes, includes multiple scattering effects, quantifies field errors and furnishes a quantitative measure of the method's accuracy, permits a systematic procedure for obtaining higher-order terms in the iterative solution of the scatter problem, and satisfies reciprocity using only the first-order solution. The first-order solution of this new full wave method has been shown to reduce to the small perturbation and the Kirchhoff approximation in their regions of validity. It has also been numerically applied to surfaces with Gaussian height and slope variations and shown to be more accurate than the small-perturbation and the Kirchhoff methods in regions where neither are considered valid. This paper extends the theory to the more general and important case of scattering by a dielectric interface, where one of the two halfspaces is lossy.

Index terms—

Felix Schwing ? , Gerald Whitman ? & Henry Tsai ? Abstract-A new full wave theory for scattering by one dimensional perfectly conducting rough surface has been formulated recently. It provides enhanced physical insights into rough surface scattering processes, includes multiple scattering effects, quantifies field errors and furnishes a quantitative measure of the method's accuracy, permits a systematic procedure for obtaining higher-order terms in the iterative solution of the scatter problem, and satisfies reciprocity using only the first-order solution. The first-order solution of this new full wave method has been shown to reduce to the small perturbation and the Kirchhoff approximation in their regions of validity. It has also been numerically applied to surfaces with Gaussian height and slope variations and shown to be more accurate than the small-perturbation and the Kirchhoff methods in regions where neither are considered valid. This paper extends the theory to the more general and important case of scattering by a dielectric interface, where one of the two halfspaces is lossy.

1 I.

Formulation of Problem by use of CC-Method Subject of this paper is a new theory of scattering from rough dielectric surfaces of the type shown in Fig 1. The interface between an air halfspace and a dielectric halfspace is rough over a length $2L$ and planar beyond this region. The roughness profile is one-dimensional, i.e. the local height D varies with z but is constant with y .

A plane wave is incident under the angle θ upon the interface; it may be incident from above or below. This incident wave and the resulting scattering field are TE-polarized, i.e. these fields consist of components E_y , H_x and H_z .

The technique used in this paper to formulate the problem is the Correction Current (CC) method, a new full wave method which has recently been established for plane wave scattering from metal surfaces [1] and subsequently extended to dielectric surfaces, with the lower halfspace at first assumed to be lossless but in the work presented here allowed to be lossy 1.

Author ? : US Army CERDEC Fort Monmouth, NJ. Author ? : NJIT Newark, NJ. e-mail: gerald.whitman@njit.edu The CC-method defines a primary field and a complete system of scatter fields, called

radiation modes. Each of these fields consists of an incident, reflected and transmitted plane wave. These component waves, however, are modified such that each of the radiation modes as well as the primary field satisfies the boundary conditions at the interface, individually and rigorously. Since these fields are simple in structure (i.e. consisting of suitably modified plane waves) and comply with the boundary conditions at the rough interface, they will no longer satisfy Maxwell's equations 1. This is remedied by introducing fictitious current distributions, called passive currents that are assumed to be associated with each of these fields and chosen such that they exactly cancel the field errors of these fields; the passive current distribution of course varies from mode to mode. Thus, while not being solutions of Maxwell's source free equations, the primary field and radiation modes are solutions of Maxwell's equations with sources 2. The passive currents exist only in the corrugation region $|Z| \leq L$; outside this region, where the interface is planar, the primary field and radiation modes are exact solutions of Maxwell's source free equations and no correction is needed.

2 S Air Halfspace

$z \geq 0$

3 2L Dielectric Halfspace

In addition, it is assumed that each radiation mode is generated by an "active" sheet current distribution residing in a plane $z = z_0$ termed the reference plane of this mode. These reference planes exist in the corrugation region $|Z| \leq L$ only, and all active currents of the mode system are limited to the same region as the passive currents. An important feature of these active currents is that taken together they form a complete orthogonal system so that any (passive) current distribution in the region $|Z| \leq L$ can be expanded into, or nullified by the active currents.

The total field is then written as a superposition of the primary field and the radiation modes. This combined field, of course, is not allowed to contain any active or passive current distribution, and this "zerocurrent" condition is then used to determine the unknown amplitudes of the radiation modes by an iterative procedure which in, a step-by-step fashion, eliminates the passive currents of the primary field and the radiation modes by the active currents of the radiation modes. The completeness, mentioned above, of the active currents allows us to do this consistently. The resulting combined field will satisfy the boundary conditions at the interface and the radiation condition at infinity while all active and passive currents are eliminated by mutual compensation. Thus the combined field is the solution of the scatter problem. For numerical efficiency, the iterative process is typically cut-off after the first iteration.

Using this approach the dielectric surface scatter problem has recently been solved for the case that the dielectric halfspace is loss free. Scattering patterns have been obtained in the form of single integrals over elementary functions which are easily evaluated numerically. The patterns have been computed for both deterministic and random rough surfaces. Comparison with the corresponding patterns obtained by a Method of Moments (MoM) procedure has shown that the first order iteration theory is of good accuracy over a wide parameter region. A paper reporting on this study, which was conducted at CERDEC and NJIT, is close to completion and will be submitted for publication in the near future.

The extension of the theory to lossy dielectrics is currently pursued under the 2007 In-house Laboratory Independent Research (ILIR) program. This extension is not straight forward, however. The problem geometry consists of two halfspaces; and two groups of radiation modes are required for full characterization of the problem. This holds for the lossy as well as for the lossless case. But when these two mode groups for the lossy case are defined in direct analogy to the two mode groups for the lossless case one of the groups diverges, i.e. increases exponentially with $|x|$, which is unacceptable. Hence the radiation modes need to be redefined, which has a significant effect on the theory. In the development of this theory it is consistently assumed that $[r]$ is finite; it may be small, but is not allowed to be arbitrarily small. Comparison to the results of the lossless theory will be made only after all formulas for fields and radiation patterns have been fully established.

4 II. Analytical Results and Discussion

In the following it is assumed that the incident plane wave is situated in the upper (air) halfspace. If it is located in the lower halfspace, the expressions obtained for fields and patterns will be different, of course. But the overall trends observed are similar.

It is interesting to note that the scatter fields associated with the two locations of the incident wave are related by a simple symmetry relation. Assume that a plane wave of amplitude E_0 and phase constant (in the z -direction) incident in the air halfspace -generates a scatter field $E_s(x, z; k_0, k_z, D(z))$. Assume, furthermore, that a plane wave of amplitude E_0 and phase constant ϕ_0 -incident in the dielectric halfspace -creates a scatter field $E_d(x, z; k_0, k_z, D(z))$. Then these two scatter fields are related by the condition.

$$E_s(x, z; k_0, k_z, D(z)) = E_d(x, z; k_0, k_z, D(z))$$

This relation holds for all x and z , i.e. in both halfspaces. Thus, if $E_d(x, z; k_0, k_z, D(z))$ has been determined $E_s(x, z; k_0, k_z, D(z))$ follows by a simple substitution of coordinates and parameters 3; and vice versa.

Symmetry relation (1) must be satisfied for the exact scatter fields. But it is satisfied already for the first order iteration approximation used in this paper which may be taken as an indication that the first order method is of good accuracy.

significant magnitude only in a narrow region adjacent to the interface, while this field, whether generated by the first or the second mechanism mentioned above, will decrease rapidly with increasing distance from the interface. The general representation of the scatter fields in terms of two double integrals (i.e. a representation akin to eq. (2a)) allows to quantify this, but the analytical procedure is rather lengthy and tedious, and not included in this paper.

One last remark: The scatter pattern (3b) in the upper halfspace includes a factor $\cos \theta$ and will be zero for $\pm 90^\circ$, i.e. at the interface, indicating that E_y along the y direction is zero, $\sin \theta = 0$, $\cos \theta = \pm 1$ for $\theta = 0^\circ, 180^\circ$, $\sin \theta = \pm 1$, $\cos \theta = 0$ for $\theta = 90^\circ, 270^\circ$. The interface declines faster than

The scatter field in the direct vicinity of the interface can be determined by an asymptotic evaluation of (2) for i.e. in a region where z is large but x remains small compared to z (Fresnel Region). One obtains The formula shows that E_y near the interface decrease with , i.e. the energy density will decrease here with -3 rather than with -1 , the rate of decrease for $|\theta| < 90^\circ$. With x , the field below the surface decreases exponentially, indicating a transmitted wave while above the surface it varies linearly with x indicating the interaction between an incident and a reflected wave. All this is consistent with a continuous leakage of energy from the air halfspace into the lossy dielectric halfspace, i.e. with the second mechanism mentioned above for generating the scatter field in the lower halfspace.

As mentioned earlier, the discussion and formulas presented in this paper apply to the case that the primary wave is incident in the upper (air) halfspace. If the antenna generating the incident wave is situated in the lower (dielectric) halfspace, the symmetry relation (1) applies; the formulas are analogous; and similar overall trends are observed.

The theory for the lossy dielectric case summarized in this paper has been developed under the 2007 CERDEC ILIR program in cooperation with NJIT and is near completion, though some points need further investigation. Final results will be tested by comparison to data obtained by a MoM technique. This work is schedule for 2008.

Numerical Techniques such as the MoM and FDTD methods can be relied on to provide very accurate results. Analytical methods as the one presented in this paper, even though approximate, have the advantage of showing parameter dependencies explicitly, thus providing physical insight. In addition, the field and pattern formulas -often obtained in the form of single integrals over elementary functions -are amenable to efficient computer evaluation and may be useful for real-time modeling.



Figure 1: Fig. 1 :



Figure 2: Fig. 2 :



Figure 3:

¹For a review of established theoretical methods for the analysis of rough surface scattering see[2]-[4].
Alternatively, the passive currents may be interpreted as quantifying the field errors.

²Note that since $\gamma = (\gamma_1, \dots, \gamma_n) \in \mathbb{R}^n$ the parameter substitution replaces γ by $1/\gamma$.

³Except when a (u-u')⁻¹ singularity occurs on the real u-axis. In this case the u-integration bypasses the singularity by a local deformation of the integration path into the positive imaginary u-halfspace.

⁴No such energy transfer occurs when γ is real, since the energy density in both half spaces decreases with $|\gamma|^{-1}$.

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