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Time-Dependent Learning Effect and Deterioration on Single Machine's Scheduling

By Dian Ayu Nurul Ihsani, Sunarsih & Robertus Heri
Diponegoro University, Indonesia

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Time-Dependent Learning Effect and Deterioration on Single Machine's Scheduling

Dian Ayu Nurul Ihsani [α], Sunarsih [σ] & Robertus Heri [ρ]

Abstract- Learning effect and deterioration do not always occur separately. If both of them are founded simultaneously, the actual processing time of the jobs will both increase and decrease at the same time. The actual processing time is defined by a function of the starting time and position of jobs in the sequence. In this paper, the effect of learning and deterioration is applied to single machine's scheduling problem in a paper-mill. Learning effect as a result of regular performance-evaluation reduce the effect of deterioration up to 206, 5509 hours. This paper-mill operates jobs by their interest. This paper show that Earlier Due Date (EDD) rule construct a better sequence under maximum lateness problem then either Shortest Processing Time (SPT) rule or Most Urgent Job rule do. Maximum lateness of the jobs under EDD rule is 13,6% less then sequence that is recently used in that paper-mill.

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I. Introduction

On-time product is needed by an industry manufacture to make a grade. Plan a good schedule is one of competitive strategy to solve that. It orders to accommodate all of the jobs in some machines and get an optimum result. Relatively to delays of production, this paper proposes maximum lateness problem on a single machine's scheduling. Maximum lateness problem is optimum when the sequence of jobs is giving smallest value of maximum lateness.

In single machine environment, scheduling is putting in order to make a sequence of jobs because the processing times are assumed to be fixed. The processing time of jobs is considered have a constant value. However, in many real situations, the company most certainly gets some factors that is make the actual processing time being longer because of deterioration or shorter because of learning.

The effect of learning and deterioration in the scheduling problem has often been learned for this recent years. Kou and Yang [5] introduce the impact of time-dependent learning effect in the single machine's scheduling that is optimum when it solved by Shortest Processing Time (SPT) rule. Koulamas and Kyparisis [4] showed that $O(n \log n)$ (n is the number of jobs) SPT rule is optimum for make span and minimizing total completion time in single machine environment if the learning effect is function of the sum of processing times that was done. Browne and Yechiali [1] are analyzed the effect of deterioration for optimum scheduling order to make a minimum makespan. Cheng, et al. [2] used $O(n \log n)$ algorithm to solve job-dependent deterioration problem in single machine under the case of due date, earliness, and tardiness.

Both of learning effect and deterioration are not always separately occur. In many real situation, both of them right usually simultaneously found. Sun [9] introduced deterioration and learning effect that is occurring in single machine at the same time. He showed that makespan, total completion time, sum of quadratic job completion time, total weighted completion time, and maximum lateness problems have an optimum solution. Yang and Kuo [12] introduced two kind of learning effect that were occurring in a deteriorated single machine. They were job-dependent learning effect and job-independent learning effect. Low and Lin [6] show that time-dependent learning effect in deteriorated single machine and flowshop environment has an optimum solution for makespan, total completion time, and weighted completion time problems.

Model that is considering in this paper is applied in a paper-mill that is use single machine to produce papers. Deterioration occurs when the machine either producing less quantity (tonase) of reel then usual or different gramatur (/) value. It gives occasion to increase the actual processing time. To solve this problem, the employers have to upgrade their capability of making paper-pulp and repair the splitting reel. Their effort to reduce the impact of delay is example of learning effect in this paper-mill. They usually sequences jobs by their weight. This sequence brings some jobs getting late. Because of that, the maximum lateness problem able to applied in this paper-mill.

II. Consideration Method

Consideration method that is used in this paper is literature-study from journals, books, and any other references. Validation of the theory in the real situation gets by applied model to existing data observation in a paper-mill. We consider three rules of scheduling to compared, that are: Earlier Due Date, Shortest Processing Time, and Most Urgent Job-sequence.

Author α: Mathematics Department, Faculty of Science and Mathematics, Diponegoro University, Prof H. Soedarto SH street, Tembalang, Semarang, Central Java 50275 Indonesia. e-mail: dienihsani@gmail.com
Author σ ρ: Mathematics Department, Faculty of Science and Mathematics, Diponegoro University, Prof H. Soedarto SH street, Tembalang, Semarang, Central Java 50275 Indonesia. e-mails: narsih_pdil@yahoo.com robertusutomo@yahoo.co.id

III. Result and Consideration

a) Actual Processing Time Model

Single machine is simplest case of scheduling because the operation is only occurring in one machine. Still single machine scheduling most often practically occur [3]. Single machine is able to be a special case of any other machine environment. Problem solving in this case is usually able to be heuristic-base of more other complex environment [8].

Learning effect is one of the impacts of effort the company to upgrade their performance. This acquisition often comes from regular either employers or overall company evaluation. An employee will find the way to do his job efficiently along his number of repeating [5]. In other world, he will do better and better along his experiences. The actual processing time of jobs is enable to calculate if the scheduled processing time unknown. This is called time-dependent learning effect in literature.

Deterioration is a condition when machine's performance piecemeal goes down. Machine is in a highest performance at the beginning. Its reducing of performance is come in sight at its longer time to completing next jobs [1]. If deterioration occur in the machine, allover of job's processing times increase under this condition. Every job get same deterioration rate because they operate in one single machine [2].

If both of learning and deterioration are occurring simultaneously, actual processing time of jobs is defined as a related function of starting time to its position in the sequence. Low and Lin [6] introduce actual processing time model for scheduling problem with time-dependent learning effect and deterioration.

There are n jobs to operate in a single machine. The machine is able for one job one time and no idle time allowed up to the last job leave the machine. Actual processing time of job that is start at time t and scheduled in position r is defined by:

$$P_{jr}(t) = P_j \left(1 - \frac{\sum_{l=1}^{r-1} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^{r-1} + st$$

$$= P_j \left(\frac{\sum_{l=r}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^{r-1} + st \qquad (3.1)$$

Where, $r = 1, \ldots, n$ with

a	learning index $(a \geq 1)$,
b	: learning index $(0 < b < 1)$,
s	: deterioration rate $(s \geq 0)$,
t	: starting time (hour),
r	: position of job in sequence,
$P_{jr}(t)$	: actual processing time of job J_j that is t started at time and scheduled in position- in sequence (hours),
P_j	: processing time of job (hours),
$P_{[l]}$	: processing time of job that is scheduled in position- in sequence (hours).

In the model above seems that the actual processing time can not calculated if processing time of the previous job unknown. Not like position-dependent learning effect, in this model r shown the number of previous jobs. Increasing number of finished job will reduce the value of $\left(\frac{\sum_{l=r}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a$. So that the processing time less steeper. In other word, the efficiency increase by number of previous job proportionally [4].

Deteriorating jobs have constant-dependence to their position [7]. Equation (3.1) satisfies the principal of learning effect by actual process capacity because dependences to position yet time at all [6].

In this model, show the relation of deterioration and job's starting time. It classically describes times increasing of starting time proportionally by its deteriorating rate.

b) Completion Time

Given schedule $\pi = [J_1, J_2, \ldots, J_n]$ with $P_{[j]}$ as normal processing time of J_j, then processing time of the first job $(j = 1, r = 1)$ in beginning position $(t = 0)$ and initial value $C_{[0]} = 0$ is

$$C_{[1]} = 0 + P_{[1]} \left(\frac{\sum_{l=1}^{n} P_{[1]}}{\sum_{l=1}^{n} P_1}\right)^a b^0 + s.0 = P_{[1]}$$

The second job $(j = 2, r = 2)$ is start after the first job finish. So that it has second value $t = C_1$ and so on. Substitute t to equation (3.1):

$$C_{[2]} = C_{[1]} + P_{[2]} \left(\frac{\sum_{l=2}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^{2-1} + s.C_{[1]}$$

$$= P_{[1]} + P_{[2]} \left(\frac{\sum_{l=2}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b + s.P_{[1]}$$

$$= P_{[1]}(1+s) + P_{[2]} \left(\frac{\sum_{l=2}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b$$

Generally, completion time of job-k $(j = k)$ is

$$C_{[k]} = P_{[1]}(1+s)^{k-1} + P_{[2]}(1+ s)^{k-2} \left(\frac{\sum_{l=2}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b + P_{[3]}(1+ s)^{k-3} \left(\frac{\sum_{l=3}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^2 + \cdots + P_{[k]} \left(\frac{\sum_{l=k}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^{k-1}$$

$$= P_{[1]}(1+s)^{k-1} + \sum_{i=2}^{k} P_{[i]} (1+ s)^{i-2} \left(\frac{\sum_{l=i}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^{i-1}$$

Where as, for $j = k+1$:

$$C_{[k+1]} = P_{[1]}(1+s)^{k-1} + P_{[2]}(1+ s)^{k-2} \left(\frac{\sum_{l=2}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b + P_{[3]}(1+ s)^{k-3} \left(\frac{\sum_{l=3}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^2 + \cdots + P_{[k]} \left(\frac{\sum_{l=k}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^{k-1} + P_{[k+1]}(1+s)^{-1} \left(\frac{\sum_{l=k+1}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^k$$

$$= P_{[1]}(1+s)^{k-1} + \sum_{i=2}^{k} P_{[i]} (1+ s)^{i-2} \left(\frac{\sum_{l=i}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^{i-1} + P_{[k+1]}(1+s)^{-1} \left(\frac{\sum_{l=k+1}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^k$$

$$= P_{[1]}(1+s)^{k-1} + \sum_{i=2}^{k+1} P_{[i]} (1+ s)^{i-2} \left(\frac{\sum_{l=i}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^{i-1}$$

Because of the equation for $j = k+1$ right, by mathematical induction, this equation is valid for every $k \in N$. Completion time of all job $(j = n)$ can wrote as

$$C_{[n]} = P_{[1]}(1+s)^{n-1} + \sum_{i=2}^{n} P_{[i]} (1+ s)^{i-2} \left(\frac{\sum_{l=i}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^{i-1} \qquad (3.2)$$

$$1 \left| P_j \left(\frac{\sum_{l=r}^{n} P_{[l]}}{\sum_{l=1}^{n} P_l}\right)^a b^{r-1} + st \right| L_{max} \qquad (3.3)$$

Problem

Optimum solution for lateness problem in single machine environment is building on this Theorem 3.1 and corollary 3.1 below.

c) Theorem 3.1

For $1|P_j(\alpha(t)+\beta r^a)|L_{max}$ problem, if the jobs have agreeable weight $(P_i \le P_j)$ implies that $d_i \le d$,, for each J_i and J_j, optimum schedule is got by sequencing jobs in non-decreasing order d_j (EDD rule) [10].

d) Proof

Let S_1 as schedule of two contiguous jobs J_i and J_j without EDD rule. In other word $d_i > d_j$ implies $P_i \ge P_j$ then $S_1 = \{\pi_1, J_i, J_j, \pi_2\}$. Another schedule is got by exchange the sequence of job-i with job-j . π_1 and π_2 are a partial sequence that is may empty. This exchange makes a new schedule $S_2 = \{\pi_1, J_j, J_i, \pi_2\}$.

Assumed there are $r-1$ jobs in π_1 where completion time of this last job is B. J_i and J_j are job-r and job-(r + 1) in S_1. Similarly, J_j and J_j are job-r and job-(r + 1) in S_2.

Completion time of S_1 under condition $P_{jr}(t) = P_j(\alpha(t)+\beta r^a)$ is:

$$C_i(S_1) = B + P_i(\alpha B + \beta r^a)$$

$$C_j(S_1) = B + \beta P_i r^a + P_i \alpha B + P_j \alpha B + P_i(\alpha B + \beta r^a) + \beta P_j (r+1)^a$$

Completion time of S_2:

$$C_j(S_2) = B + P_j(\alpha B + \beta r^a)$$

$$C_i(S_2) = B + \beta P_j r^a + P_j \alpha B + P_i \alpha B + P_j(\alpha B + \beta r^a) + \beta P_i (r+1)^a$$

So, the lateness:

$$L_i(S_1) = B + P_i(\alpha B + \beta r^a) - d_i$$

$$L_j(S_1) = B + \beta P_i r^a + P_i \alpha B + P_j \alpha B + P_i(\alpha B + \beta r^a) + \beta P_j (r+1)^a - d_j$$

$$L_j(S_2) = B + P_j(\alpha B + \beta r^a) - d_j$$

$$L_i(S_2) = B + \beta P_j r^a + P_j \alpha B + P_i \alpha B + P_j(\alpha B + \beta r^a) + \beta P_i (r+1)^a - d_i$$

Let $L(S_1)$ is lateness of job-(r+ 2) in S_1 and $L(S_2)$ is lateness of job-(r+ 2) in S_2, can be seen that $L(S_1) = L(S_2)$. Let $L_i(S_1)$ and $L_k(S_1)$ are lateness of J_i and J_k in S_2. Then $L_i(S_2)$ and $L_k(S_2)$ lateness of J_i and J_k in S_2 . Maximum lateness of S_1 is

$$L_{max}(S_1) = \max\{L(S_1), L_i(S_1), L_k(S_1)\}$$

Then maximum lateness of S_2 is

$$\mathrm{L_{max}(S_2) = \max\{L(S_2), L_i(S_2), L_k(S_2)\}}$$

Because of P_j always positive, then $L_j(S_1) > L_j(S_2)$ Meanwhile, because $d_i > d_j$ and $P_i \ge P_j$, , then $L_j(S_1) > L_i(S_2)$. Therefore,

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$$L_k(S_1) > max\{L_i(S_2), L_k(S_2)\}$$

$$max\{L(S_1), L_i(S_1), L_k(S_1)\} >$$

$$max\{L(S_1), L_i(S_2), L_k(S_2)\}$$

$$\text{with } L(S_1) = L(S_2)$$

$$max\{L(S_1), L_i(S_1), L_k(S_1)\} >$$

$$max\{L(S_2), L_i(S_2), L_k(S_2)\}$$

$$L_{max}(S_1) > L_{max}(S_2)$$

$L_{max}(S_1) > L_{max}(S_2)$ shows that S_1 is not optimum schedule because there is another schedule make smaller maximum lateness. It proofs that sequencing jobs by non-decreasing order of d_j reduce maximum lateness (EDD rule).

e) Corollary 3.1

For $1 \quad j \quad \frac{\sum_{l=r} P_{[l]}}{\sum_{l=1}^{n} P_l} \quad b^{r-1} + st \quad max$ problem, if the jobs have agreeable weight $(P_i \leq P_j)$ implies that $d_i \leq d_j$, for each J_i and J_j optimum schedule is got by sequencing jobs in non-decreasing order d_j (EDD rule).

f) Case Simulation in Manufacturing

A paper-mill that is has deterioration rate expect S = 0,004 to produce 74 tons paper daily (about 2.250 tons for a month). But then, on May 2014 they only produce 2.188 tons along that month. From the existing data of processing time of 159 jobs in May 2014, gotten index of learning effect $a = 1{,}4$ and $b = 0{,}99$.

Normal processing time by the data is 700.16 hours. Though learning is qualitative, the impact of this effort can expect by the production data. Effect of learning in this paper-mill by existing data on May 2014 is shown in Table 3.1.

Table 3.1 : Comparison of Completion Times (hours)

Normal	$s = 0.004$	$s = 0.004$, $a = 1.4, b = 0.99$
700.16	926.4909	719.94

Although jobs still delay 19.76 hours, but effect of learning in this paper-mill reduces the impact of deterioration up to 206.5509 hours. This paper-mill usually sequence jobs by its weight. They make a mark of each job and operate job with highest mark first. This sequence is similar with Donald Waters's [11] scheduling rule, Most Urgent Job First (MUJ).

Furthermore, to determine optimum sequence under the maximum lateness problem, there are three rules of scheduling will be compared. There are EDD, SPT, and Most Urgent Job that is recently used in this paper-mill. The output of this comparison is written in Table 3.2.

Table 3.2 : Comparison of EDD's, SPT's, and MUJ's Output

	MUJ	SPT	EDD
Late-product (unit)	65	59	59
L_{max} (jam)	1859,2	1735,678	1606,856

Table 3.2 show that EDD rule gives smaller value of maximum lateness. If it was compare to MUJ that is recently used in this paper-mill, EDD less the value of maximum lateness up to 13.6%.

IV. Conclusion

Actual processing time model for single machine with time-dependent learning effect and deterioration is applicable to Earliest Due Date (EDD) rule. In the case simulation in a paper-mill, sequence under EDD rule give smallest maximum lateness then either Shortest Processing Time or Most Urgent Job First does. The maximum lateness of EDD rule less 13.6% then Most Urgent Job First rule that is recently used in this paper-mill.

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