

# A Novel Approach for the design of 2D Sharp Circularly Symmetric FIR Filters

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## Abstract

In applications like weather monitoring, remote sensing, seismic signal processing etc, the boundaries and sharp features of the images have to be enhanced to obtain useful results and interpretations. The two dimensional(2D) highpass filters are used for such image processing applications. A direct design of the 2D high pass filter using frequency transformation has not been reported in the literature so far. In this paper, we present the design of a sharp 2D high pass filter using a direct transformation of the 1D low pass filter into a 2D high pass filter. Chebyshev polynomial approximation is used in mapping the one dimensional (1D) filter into its 2D equivalent. A promising aspect of the proposed transformation is that it is multiplier-free. Therefore the total complexity is decided by the multipliers of the 1D prototype filter. The complexity of designing a sharp 1D filter can be reduced by using the technique of Frequency Response Masking (FRM). Better circularity of the contour in the cutoff radius of the 2D filter is achieved compared to the traditional McClellan transformation. Reduced computation time and complexity are the highlights of the proposed approach.

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**Index terms**— Multiplier-less transformation, 2D sharp FIR Filter, Frequency Transformation, Circular symmetry, Frequency Response Masking.

## 1 Introduction

ver the last few decades, the area of the design and implementation of 2D filters has attracted much attention of the researchers owing to its wide deployment in various areas like biomedical signal processing, seismic signal processing, radar and sonar procesing, genomic signal processing, satellite image processing etc ??Williams et.al.,2007 ?? Boudjelaba et.al.,2011). The various methods for the design of 2D linear phase FIR filter include frequency sampling, windowing, and frequency transformation ??Lim,1990). Although the first two methods produce a better approximation of the response to the ideal one, the filter design involves large computational efforts especially for higher order filters. In contrast, the McClellan transformation has been bestowed with the features of reduced computation time and lesser implementation complexity. The higher order designs of the 2D zero phase filter can be done with much less computation efforts ??Merserau,1980). The above features have contributed to lot of significant research performed in this area and very recently, this technique has been extended to the design of 3D filters ??Mollova and Mecklenbrauker,2009).

Two dimensional high pass filters are extremely useful in detecting the sharp edges and boundaries of a given image. For instance, in weather monitoring, the discrete 2D high-pass filters are used to isolate the boundaries and edges of the weather data to make appropriate inferences and predictions ??Feser and Storch,2005). Besides, high order high pass filter is shown to be a useful tool to evaluate and interpret meteorological data in finite areas in the work by ??aymond,1989. The 2D high pass filter along with its complimentary lowpass filter has also been deployed in applications like the two channel quadrature mirror filter bank and the crossover network for image coding ??Mitra and Yu,1986). There are several techniques for the design of 2D low pass filters ??Lim,1990) and hence in most cases, the high pass filter is obtained using the complementarity property (Mitra and Yu,

1986). Hence the direct design of high pass filters has not been addressed properly. In this paper, we propose a novel method for the direct design of 2D high pass filter. To this end, we use an approach similar to that is used in McClellan transformation using Chebyshev polynomial approximation for mapping the 1D filter into its 2D equivalent. But the striking difference is that the proposed transformation is used to derive a High Pass 2D filter directly from the 1D low pass prototype filter. Such an approach has not been reported in the literature so far. An advantage of using this approach is that this method preserves most of the characteristics of the 1D filter, especially the transition width and ripple characteristics. Once the high pass 2D filter is obtained, the complimentary low pass filter can be derived. This technique also avoids the drawback of the traditional McClellan transformation giving squarish contours at wide-band cutoff radii. The above transformations being multiplier-less, the number of multipliers needed for the realization of the 2D filter is decided by the order of the 1D FIR filter. Hence the number of multipliers of the 2D filter is fully decided by that of the 1D prototype filter. performance specifications are close to an ideal 2D filter, the one dimensional prototype filter should be sharp. Even though 1D linear phase FIR filters are acclaimed for their inherent stability, computational complexity becomes exceedingly large as the transition width is reduced. To circumvent this drawback, we utilize the Frequency Response Masking (FRM) approach (Lim,1986) for the design of the sharp 1D prototype low pass filter and it results in sparse coefficients. FRM technique offers significant savings in the number of multipliers compared to the traditional mini-max design for the design of 1D filter. Due to the enormous computational saving possible for the design of an FRM filter, it has been extended to a variety of applications like array beam forming, FPGA, audio processing etc (Lim.et.al.,2007). Thus the 1D filter designed using FRM is converted into the 2D high filter using the proposed transformation. In many of the previous work using McClellan transformation, either optimization procedures or numerical methods were performed to obtain good circularity of the 2D radius. But in the very recent work by Liu and Tai, 2011, none of the above techniques were needed to obtain circular contours even at wideband. Instead a cascading term was added to the traditional McClellan transformation and two transformations namely T1 and T2 were proposed in the above paper. Since an analytical expression has been derived, the contour mapping problem is avoided in the T1 and T2 transformations. Our approach for the design of circular symmetric filters is based on the derivation of a totally new transformation instead of the mapping function in the traditional McClellan transformation. The contour mapping problem is avoided in our proposed method also. Hence the computation time is reduced drastically. In short, the sharp 2D filters designed using the proposed technique is better for high speed and low power applications while offering good performance close to the ideal one. This paper is organized as follows. Section II deals with an overview of frequency response masking approach. In Section III, the proposed transformation is briefed. The design of the 1D FRM filter is explained in Section IV A. Section IV B addresses the proposed design of the 2D circularly symmetric sharp filters. Simulation results are presented in Section V and conclusions are made in Section VI.

## 2 II.

### 3 Overview of Frequency Response Masking

Frequency Response Masking is a much acclaimed technique for the design of an arbitrary bandwidth sharp FIR filter with reduced computational complexity. Efficient hardware implementation of the filter designed using FRM is possible due to the large number of zero valued coefficients. It consists of a prototype filter , complimentary filter , masking filter and a complimentary masking filter . The complimentary filter can be realized by subtracting the output of the low pass filter from the delay block , provided, the model filters The overall transfer function of the FRM filter can be written as follows (1) where

The prototype filter, also called the band edge shaping filter, is interpolated by a factor M and therefore its transition width is reduced by a factor of M. The masking filters are used to retain the necessary spectrum repetitions for the formation of any arbitrary bandwidth filter under consideration. obtain a 2D low pass filter. High pass filter was derived from it using the complementarity property (Mitra and Yu,1986) The contour mapping of the proposed transformation is shown in Figure 2. A promising feature of the proposed design of the 2D filter is that the contour mapping problem is avoided in our work, since, an analytical expression is available for the proposed transformation. Hence, the 2D filter is obtained by the direct application of the proposed transformation. This calls for reduction in the computation time of the 2D filter.

### 4 III. Proposed Frequency Transformation

#### 5 b) Frequency Mapping of the proposed H1 Transformation

The relationship between the one dimensional frequency and the two dimensional radius is derived below.

### 6 Fig.2 : Contour of the proposed transformation

Utilizing the quadrantal symmetry of the proposed transformation, let us consider only one direction in the 2D plane namely the 1 axis and make  $\theta = 0$  and  $\theta = \pi$ . This is done to obtain a one to one mapping between the frequencies in the 1D and 2D.

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100 After applying the above condition, Equation.6. is modified as follows (7) From Equation.7., it can be found  
101 that the 1D frequency can be obtained as given below. (8) More clearly, the mapping is given by (9) The figure  
102 showing the relation between the 1D and the 2D frequencies is given in Figure ??3.

## 103 7 c) Realization of the proposed H1 transformation

104 In order to realize the transformation it has to be expressed in terms of  $z^{-1}$  and  $z^{-2}$ . Hence using the trigonometric  
105 identities and the transformation in Equation ( ??) can be rewritten as (10) Therefore the final transfer function  
106 of the transformation is given by (11) where and

107 After rearranging the terms, above expression can also be written as follows  $2) = = | = 1 = 1 - 2) +$   
108  $-1)(1/4 ( z z 2) + ) - 1) (1/4($

109 The block diagram of the proposed H1 transformation is shown in Figure ??4. From the realization, it can  
110 be found that the transformation is totally multiplierless. Hence the total number of multipliers of the 2D filter  
111 is decided by the multipliers of the 1D filter. It is found that the computational complexity of the proposed H1  
112 transformation, is least when compared to the T1, T2 and the McClellan transformation.

113 The generalized McClellan transformation for obtaining circular contours as given in Kidambi, 1995 is used  
114 to make the comparison of complexity. The expression for McClellan transformation in Kidambi, 1995 is given as  
115 follows. (12) The computational complexity of the transformation is found in terms of the number of adders and  
116 multipliers and the comparison is given in Table ??.

## 117 8 MC refers to generalized McClellan transformation.

118 Table1 : Complexity comparison of various transformations IV.

## 119 9 Proposed Design of the Sharp 2d

120 Circularly Symmetric Filters

121 A direct design of 2D high pass filter using frequency transformation has not been reported in the literature so  
122 far. All the work in this direction relied on the design of the 2D low-pass filter first, followed by finding out the  
123 complimentary high pass filter. The proposed design technique can be extended to the design of a 2D filter with  
124 the 1D prototype being any linear phase 1D filter. FRM has been used in our work to generate sharp filters with  
125 low complexity. Once the design specifications of the 2D High pass filter are obtained, using the relationship in  
126 Equation.9., the band edges of the 1D prototype low-pass filter are obtained. The design of the 1D prototype  
127 low pass filter using the FRM technique is briefed below.

## 128 10 a) Design of 1D FRM FIR Low Pass filter

129 The band edges of the various sub-filters of the FRM filter are obtained as given in the original work on FRM  
130 by ??im, 1986. The optimal interpolation factor M is obtained in such a way that the total number of multipliers  
131 needed for the realization of the overall FRM filter is minimum. The sub-filters namely the model filter  $H_a(z)$ ,  
132 masking filter  $H_{ma}(z)$  and complimentary masking filter  $H_{mc}(z)$  are designed as per the Remez Exchange  
133 algorithm. The conditions as outlined in the work by ??u .et.al., 2003 have been used to obtain a linear phase  
134 for the overall FRM FIR filter. As the objective of the work is to design a 2D high pass sharp filter with reduced  
135 complexity, FRM technique is used to obtain a sharp 1D filter with reduced complexity. The transformation  
136 preserves the properties of the 1D filter like the transition width, ripple characteristics etc.

## 137 11 b) Proposed Design of the 2D circularly symmetric Filters

138 A direct transformation is applied to the 1D prototype filter to obtain the sharp circularly symmetric High pass  
139 filter. A promising aspect of this design approach is that better circularity is obtained for all the contours when  
140 compared to the traditional McClellan Transformation. Also circularly symmetric 2D FRM based FIR filters  
141 have been reported for the narrow band case only.

142 Besides, a 2D circularly symmetric low pass filter can be obtained by the complementarity property as given  
143 in the work by Mitra and Yu, 1986. Here, only one adder is needed in addition to number of adders needed  
144 for the realization of the high pass filter. This low pass filter has the advantage that it has lesser complexity  
145 compared to the T1 and T2 transformations proposed very recently for the design of circularly symmetric low  
146 pass filter ??Liu and Tai, 2011). The realization of the 2D filter is shown in Figure ??5. This realization is quite  
147 similar to that used for the McClellan Transformation and the only difference being the use of a new  $F(1, 2)$   
148  $)$ . In this figure,  $a(n)$  corresponds to the coefficients of the 1D FRM FIR filter. A typical application of the  
149 complimentary pair of 2D filters is in the image coding system using the crossover network as pointed out by  
150 ??itra and Yu, 1986. In this paper, a new measure is proposed for evaluating the contour approximation error.  
151 Define  $C$  as the 1D frequency and  $r$  as the 2D radius. The error measure  $E(r)$  is briefed as the  $L_2$  norm of the  
152 error of approximation. It is given below. (13) The algorithm for finding the error is briefed below.

153 1. Fix the value of the 2D filter cutoff radius  $r$ .

154 2. Vary theta for given cutoff radius and obtain  $F(r, \theta)$ . 3. For each radius, using inverse mapping, find the  
 155 value of  $C$ . 4. Compute  $E(r)$  as per eqn.13. 5. Repeat the steps 2-4 for varying values of  $r$ . The approximation  
 156 error is tabulated in Table ??.

157 The computational complexity of a 2D high pass filter designed using the proposed H1 transformation is  
 158 shown in Table ???. Here  $2N+1$  is the length of the 1D prototype filter. To derive the high pass filter, the  
 159 complimentary property was used. Here, only one adder is needed in addition to number of adders needed  
 160 for the realization of the low pass filter. This high pass filter has the advantage that it has lesser complexity  
 161 compared to the T1 and T2 transformation proposed very recently for the design of circularly symmetric low  
 162 pass filter (Liu and Tai, 2011).

## 163 12 Simulation Results

164 Simulation is done using MATLAB 7.10.0 on a Dual Core Opteron processor. The proposed approach has been  
 165 deployed to obtain typical FIR 2D high pass filter. From the high pass filter, the complimentary low pass filter  
 166 can also be obtained. Two design examples have been illustrated to demonstrate the usefulness of the proposed  
 167 approach. One wideband and one narrowband high pass filter example have been taken and the equivalent low-  
 168 pass filter was found out in each case. The narrowband 2D high pass example is given as Case1. The wideband  
 169 2D high pass filter is given as Case 2.

## 170 13 Case 1

171 The band-edges of the 1D prototype filter to be designed are found as  $p=0.5884$ ,  $s=0.6101$ .

172 The magnitude response of the 1D prototype filter using FRM technique is shown in Fig. ?? The band-edges  
 173 of the 1D prototype filter to be designed are found as  $p=0.7933$ ,  $s=0.8086$ . The magnitude response and  
 174 contour of the 2-D high pass filter for Case 2 is shown in Fig. 11 and 12 respectively. Fig. 13 shows the magnitude  
 175 response of the complimentary low pass filter for Case 2.

## 176 14 Conclusion

177 A novel and direct design approach for obtaining the 2D high pass, circularly symmetric, zero phase filter using  
 178 the frequency transformation method has been presented. This transformation is then applied to a sharp 1D  
 179 filter designed using Frequency Response Masking(FRM) technique. FRM is used to reduce the computational  
 180 complexity of the sharp filter. From the 2D high pass filter, the complimentary low-pass filter can also be derived.  
 181 The promising features of the proposed design technique is the reduced computation time and reduced complexity  
 182 compared to the traditional methods. Good circularity is obtained at both narrowband and wideband radius of  
 183 the 2D filter compared to McClellan transformation. Regarding the performance of the resulting 2D high-pass  
 filter, the complexity is less compared to the recently proposed T1 and T2 transformations. <sup>1</sup>



Figure 1:



1

Figure 2: Figure. 1 :



Figure 3:



Figure 4:



Figure 5:



Figure 6: A



Figure 7: A Fig. 5 : Fig. 4 :



Figure 8:

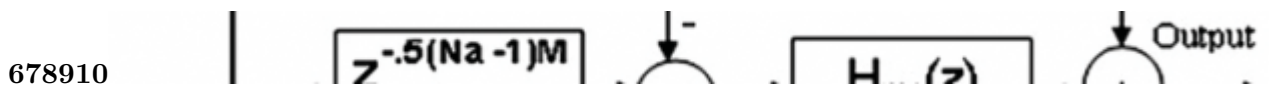


Figure 9: Fig. 6 : Fig. 7 : Fig. 8 : Fig. 9 : Fig. 10 :

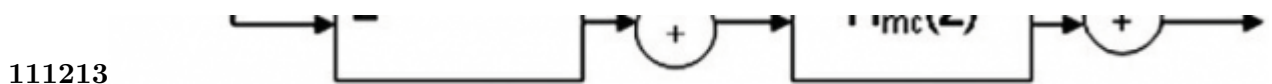


Figure 10: Fig. 11 : Fig. 12 : Fig. 13 :

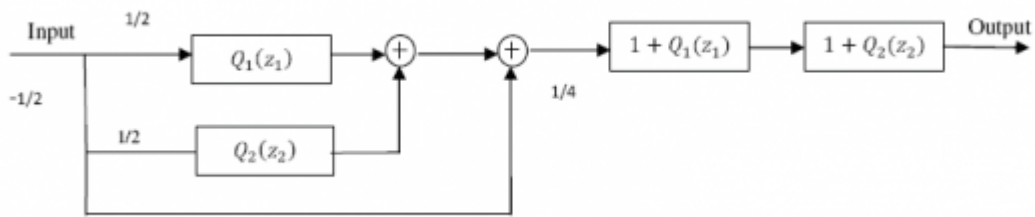


Figure 11:

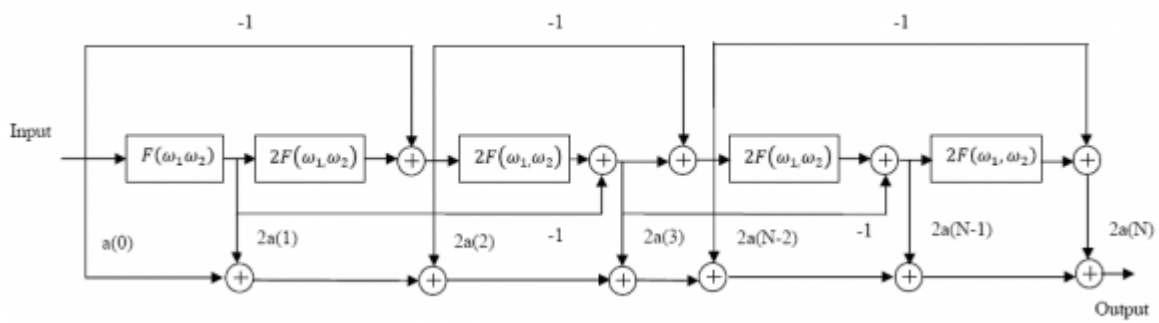


Figure 12:

3

V.

Figure 13: Table 3 :

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## 1 Appendix i

The proposed transformation for the direct design of 2D high pass filter has been found out using the Linear Least Square Estimator [Kay,1993].

Linear Least Square Estimator attempts to minimize the difference between the estimated and true parameter values. Here the unknown parameters are the coefficients of the basis functions being  $\sin^2(\omega_1/2)$ ,  $\sin^2(\omega_2/2)$  and 1 respectively. Consider the mapping given by  $S=HU$  S is the vector of dimension  $N \times 1$ , H is an  $N \times P$  matrix where N is the number of observations. P is the dimension of the vector U. X is the actual value. The least Square Estimate is found by minimizing the least squared error  $J(U)$  given by the following expression.

$$J(U) = (X - HU)^T (X - HU).$$

Setting the gradient of the the above =0, we get, the estimate of U as

In the above method, the values of S are the known values of the expected contour defined at various values of the 2D frequencies  $(\omega_1, \omega_2)$ . There are N observations. H is the matrix obtained by evaluating the values of the basis functions at various values of  $(\omega_1, \omega_2)$ . In our work, we have  $N=20$  and  $P=3$ . The estimated values of the coefficients of the constituents in the mapping are  $=0.4848$ ,  $=0.4848$ ,  $=1$ . In order to get a multiplier-less realization, and were taken to be 0.5. Hence the transformation is as follows.

But the contours of this transformation had noncircularity in the outermost radii. To correct this, a cascading term is added to the above transformation. Hence the resulting transformation was found to be as follows.

Where

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