

Conceptual Model Development of Lime Versus Cement Stabilized Expansive Soils

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Received: 15 December 2011 Accepted: 2 January 2012 Published: 15 January 2012

Abstract

This paper presents conceptual development of fundamental concepts of modelling the elasto-plastic behaviour of expansive soils stabilized soils with lime and cement. The stabilization is accomplished with both lime and cement treatments of expansive soils where lime proves to be the best additive in treatment of plastic soils than cement. The concepts of the yield surfaces of the Tresca, von-Mises, Drucker-Prager, Mohr-Coulomb and Cam-Clay elasto-plasticity models are reviewed. Because the initial consumption of lime (ICL) of 3.5

Index terms— Constitutive Models; Elastoplasticity; Cohesive soils.

1 Introduction

Like untreated soil, lime and lime -cement stabilized soil is not entirely pure isotropic, elastic and homogenous material but rather a material with elasto-plastic behaviour. To simulate the complex behaviour of both natural and treated soils, constitutive models assuming linear elastic perfectly plastic brittle weakening behaviour are assessed. Elasto-plasticity constitutive models of soil have received the attention of many writers like Gens et al. (2008), Bazant & Prager, (1985), Beer & Watson (1992), Kolar & Nemeč (1989), Merouani, (2004), Vermeer & Neher, (1999) and Chen et al. (1994). The stress strain response in these models assumes that the material a linear elastic behaviour prior to yielding and perfectly plastic behaviour after yielding. In these cases, the peak and residual strength values can be different depending on the type of soil. This study pays attention to plasticity models such as Tresca, Author : Ardhi University (ARU), Tanzania, P.O. Box 35176, Dar es Salaam, Tanzania. E-mail : charleslucian@gmail.com, lucian@aru.ac.tz von Mises, Drucker-Prager, Drucker-Prager Cap, Mohr-Coulomb, Rankine Model, Modified Cam Clay, Lade and Egg Cam-clay models. The purpose of the study therefore is to test the performance of the said soil models in the simulation of drained triaxial tests on both untreated and lime and cement treated expansive soils.

2 II.

3 Model Development Theories

Lime and lime-cement treatment or stabilization has been conventionally used in engineering to enhance the properties of expansive soils. The Lime and lime cement stabilized soils exhibit yielding behaviour when loaded. The material behaviour of soil cannot be described as a linear isotropic elastic material but a combination of elastic, plastic and viscous flow behaviors (often referred to as creep). Therefore, the two major aspects of soil behaviour, namely elastic and plastic (elasto-plastic) are under consideration in this study. The great difference between plasticity and nonlinear elasticity is that elastic deformation is fully recoverable (reversible) on unloading whereas plastic deformation is non-recoverable (permanent). The relationship between stress and strain can be presented in two forms that are strain hardening and strain softening. Normally consolidated soils and loosely packed soils are strain hardening because they tend to compress and reach a critical state when sheared. Densely packed soils and overconsolidated soils are strain softening because they tend to expand (dilate) requiring large work to overcome the interlocking as they reach critical state at large strains. In densely packed soil the hardening

5 MATERIALS EMPLOYED A) SOIL

43 appears just before the peak stress and the softening just after. On the other hand, the loosely packed soil possesses
44 strain hardening only.

45 Any material under a multi-axis state of stress will yield when the maximum shear stresses exceed the yield
46 shear strengths of the material. The plasticity theory for granular materials that include a yield surface is best
47 described by Tresca Model (Yu, 2006). Figure ?? shows Tresca model in 3-D space of principal stresses system
48 for Yield criterion. The model is often idealized for cohesionless ($c=0$) frictionless ($\mu = 0$) soils. The maximum
49 shear strength is as shown in equation 1: and k is material constant representing a yield stress in a pure shear
50 test. For failure to occur the equation above is rearranged to yield the following expression: $\sigma_2 - \sigma_3 = k$ (2)

51 For uniaxial tension: On the other hand, Von Mises postulated (1913) that a material will yield when the
52 distortional energy at the point in question reaches a critical value (Yu, 2002). Figure ?? shows a typical sketch
53 of an isotropic elastic-perfectly plastic von Mises model. The model is based on distortional energy necessary to
54 initiate yielding. Von Mises criterion incorporates the contribution of the intermediate stress to the yield state.
55 It highlights that yield occurs when the second invariant of the deviatoric stress reaches a certain value. The
56 exact solution for the von Mises yield criterion is given by the following expression: $\sqrt{\frac{1}{2}(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sqrt{3}k$ (5)

57 where: $\sigma_1, \sigma_2, \sigma_3$ are principal stresses and k is material constant (6)

58 For yielding in uniaxial tension: $\sigma_1 = \sigma_2 = \sigma_3 = 0$ (7)

59 Substituting equation 7 into equation 6, the following expression is obtained: $\sqrt{3}k = \sigma_1$ (8)

60 Substituting (8) in yield criteria (5) the following usual form of von Mises yield criterion is obtained: $\sqrt{\frac{1}{2}(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sigma_1$ (9)

62 Another failure criterion is according to Drucker-Prager Model which bases on the fact that the strain rates
63 increase with the increase in yield strength (Drucker and Prager, 1952). The model is used to modulate materials
64 that exhibit pressure-dependent yield such as soil and rocks. The model has an advantage that it handles the
65 gross inelastic coupling between deviatoric and volumetric behaviours of soils. Figure ?? shows Drucker model
66 without a cap that was later modified to the cap model (Figure ??). The Drucker-Prager Cap Model failure
67 criterion for cohesive soils (Chen, 1994) is as follows: $\sqrt{\frac{1}{2}(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} + \alpha(\sigma_1 + \sigma_2 + \sigma_3) = k$

68 and $k =$ material constants related to the friction and cohesion of soil respectively determined from the Mohr-
69 Coulomb stress invariant To capture soil behaviour in general, Mohr-Coulomb introduced an elastic perfectly-
70 plastic model to serve as a first-order model (Ti et al., 2009). Failure criterion in the Mohr-Coulomb bases
71 on the assumption that the maximum shear stress as well as principal stresses is the only measure of failure.
72 Figure 5 represents the Mohr-Coulomb yield surface in deviatoric plane while Figure 6 represents it in 2-D
73 system. The failure of the Mohr-Coulomb is the best straight-line envelope touching the Mohr's circle (Figure
74 6). Mathematically the equation for the best straight-line envelope is as follows: $\tau = c + \sigma \tan \phi$

75 (11) where:

76 τ is the shear stress, σ is the normal stress (negative in compression), c is the cohesion of the material, and ϕ is the
77 material angle of friction.

78 In terms of principle stresses, the Mohr-Coulomb failure criterion is as follows: $\sigma_1 - \sigma_3 = \frac{2c}{\sin \phi} + \frac{2\sigma_3 \tan \phi}{1 - \sin \phi}$ (12)

79 Where Another important theory is the Rankine Model which is the maximum normal strength hypothesis
80 based on similar supposition to that of Coulomb. It states that failure occurs whenever one of the maximum
81 three principle stresses equals the strength. It finds its use in ductile materials. The yield surface associated with
82 this criterion is given by: where σ_t is the tensile strength at failure. Furthermore, Roscoe and Burland (1968)
83 originally described the Modified Cam Clay Model (MCCM) to distinguish it from the earlier model called Cam
84 clay (Roscoe and Schofield, 1963 and Ortiz, Pandolfi, 2004 & Carter and Liu, 2005). The modified Cam clay model
85 employs the concept of yield criteria defined by the ellipsoid as is shown in Figure ?? . It is an elasto-plastic model
86 having non-linear elasticity characteristics prior to yielding. The model takes into account the aspect of plastic
87 volume change in compression. The model captures the commonly observed properties such as an increasing
88 stiffness as a material undergoes compression, hardening/softening and compaction/dilatancy behaviour, and
89 eventually reaching a state in which the strength and volume become constant. The model is described in terms
90 of effective stresses p and q which are very important to the area of soil response in conventional triaxial test.
91 For simplification, the failure model is simply presented in 2-D system (Figure ??). The cam clay yield rule (flow
92 rule) reads as: For Cam Clay Cap Model (Figure ??) the yield function is affected by a as follows: $f = \frac{1}{2} \left[\left(\frac{p}{M} \right)^2 + \left(\frac{q}{M} \right)^2 - 1 \right]$ (16) Failure Criterion is as follows: $f = 0$ (16) , $(p - M) \left(\frac{q}{M} \right) = 0$, $(p - M) \left(\frac{q}{M} \right) = 0$ (17)

95 And the failure criterion is as follows: As the modification of Modified Cam-clay model, the Egg Cam-clay
96 model (Figure 10) was proposed (Yu, 2002 ?? Wood, 2004 and Suebsuk et al., 2010). This model is able to capture
97 two key features namely nonlinear elasticity model and plasticity model.) are inevitable for some soil models.
98 These are easily determined in Continuum Mechanics and to save on space only final expressions are included
99 here: i. Stress Invariants $(\sigma_1 + \sigma_2 + \sigma_3)$, $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$ (19) $M = 2 \frac{c}{\sin \phi} + \frac{2\sigma_3 \tan \phi}{1 - \sin \phi}$

4 The nonlinear elasticity model demonstrates an

5 Materials Employed a) Soil

102 The soil used in this study was obtained from a 3.5 m deep open pit dug in Kibaha, Tanzania where expansive soil
103 is abundant. The soil in the area is classified as a highly expansive clay of high plasticity (Lucian, 2008(Lucian,

104 & 2009)). The maximum dry density (MDD) and optimum moisture content (OMC) for the soil in consideration
105 are in the region of 1910kg/m³ and 11.7% respectively. Some of the determined engineering properties of the
106 natural soil are summarized in the following Table ?? b) Stabilizers

107 The stabilizer materials used in this study were Lime and Cement. The cement used was the Ordinary
108 Portland Cement, Twiga brand from Tanzania Portland Cement at Wazo Hill, Tegeta, Dar es Salaam. The
109 powder hydrated lime was also obtained locally in Tanzania. The required quantity of hydrated lime was sieved
110 through No. 40 sieve before mixing.

111 6 c) Specimen Preparation

112 Air-dried soil samples for mixing were pulverized and sieved through No. 40 sieve and oven dried at 50°C for 24
113 hours. The soil was then mixed with the various amounts of the stabilizers and the required amount of water.
114 Specimens were then prepared by compaction in specimen moulds. Hydrated lime and Ordinary Portland Cement
115 were used to stabilize the samples. The initial consumption of lime (ICL) of the soil had been determined to be
116 3.5% and mellowing period to be 4 hours. Thus, 4%, 6%, 8% and 10% of lime by weight of dry soil was added to
117 the soil and cured for 7, 14 and 28 days, after which laboratory experiments were conducted. Untreated soil and
118 lime-treated samples were subjected to CU triaxial compression tests four hours after preparation (mellowing).

119 7 IV.

120 8 Experimental Results And Observations

121 The results for CU triaxial compression tests are presented in Table 1 and Figure ??1. Furthermore, Figures
122 13 -15 show effective stress Mohr circle and failure envelope obtained from triaxial test for nontreated, 4%, 6%
123 and 8% lime treated expansive soils respectively. The best fit tangent failure lines were drawn tangent to the
124 Mohr circles to show the failure envelope. For cement treated soils the Mohr-Coulomb failure turned out to
125 be a curve, therefore it was not possible to report particular strength parameters. The results indicate that
126 lime-treatment greatly improves the strength of the soil, both in terms of the internal angle of friction (from 14°
127 to 33°) and cohesion (from 17 kPa to 300 kPa) in four hours mellowing period. Further, the samples treated
128 with 6% lime show better strength properties than the other tested mix proportions. It is likely that higher
129 lime content (e.g. 8% lime) creates excess lime in the mixture that makes the sample less cohesive and weaker
130 than the lower (6%) lime-treated samples. The semi-barrelling form of failure for the 8% lime-stabilized sample
131 supports this argument, when compared with the 6% lime-treated sample which shows a clear shear form of failure
132 (closely similar to that of granular soils; ref. ??figure 11). Although perhaps adequate as a first approximation,
133 the Mohr-Coulomb criterion is the elastoplastic model of general scope with fixed yield surface, thus does not
134 accurately model the actual failure conditions of real soils. Therefore, a model whose yield surface is not fixed
135 but expands due to plastic straining to account for the plastic deformation of expansive soils is called for. When
136 the plastic deformation occurs, the yield surface changes in size, shape and degree of inclination. To capture
137 that complex behaviors of expansive soils as well as predict the true triaxial test results, the modified Cam Clay
138 Model (MCCM) is introduced (Figure 12). It can be seen

139 9 Concluding Remarks

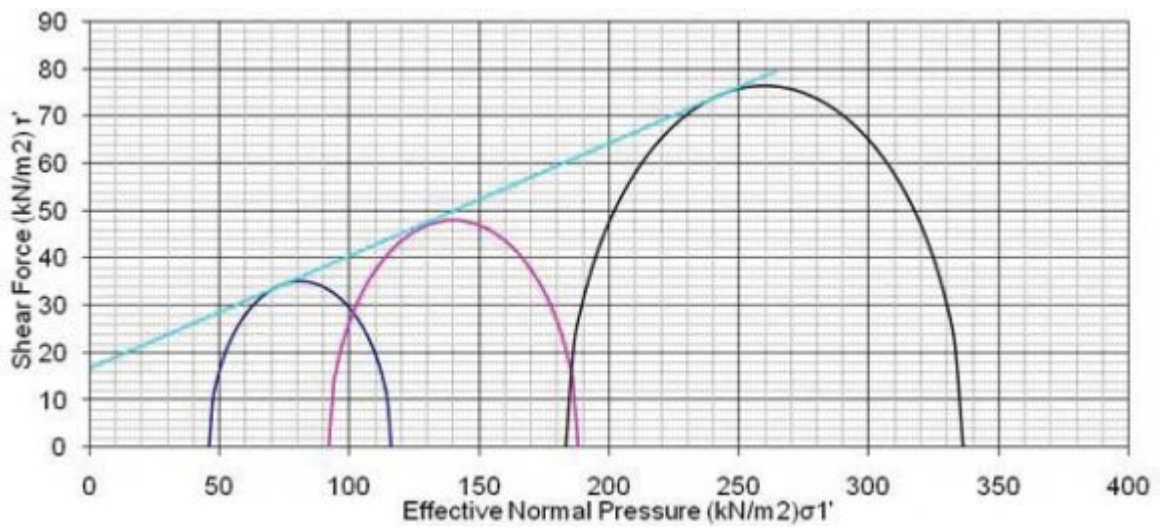
140 Practical application of models in finding solution of real-world problems attracts little theoretical or practical
141 attention from Geotechnical Engineers. Therefore, proper application of these models requires thorough
142 understanding of applications, basic features and limitations of various models. Efforts in this paper have been
143 directed to several soil models to describe the behaviour of lime vs. cement stabilized expansive soils in Kibaha,
144 Tanzania. It is obvious from the models that for the case of Theories of Shear Strength and Deformation, the
145 Mohr-Coulomb failure criterion pays no attention to strain which accompany soil failure at peak strength. On the
146 other hand the Von-Mises criterion is typically applicable for elastic plastic material. However, it enjoys superior
147 level of acceptance for friction behaviour of idealized undrained frictional cohesive material like sand. Rankine
148 model is the best fit for brittle materials. Mohr-Coulomb and Drucker-Prager elasto (visco)-plastic models are
149 typically for soils and other frictional materials. However, the Mohr-coulomb model neglects the effect of the
150 intermediate stress, 2 but the Drucker-Prager takes it into account. The Drucker-Prager, however, overestimates
151 the strength of soil. The Tresca Model is ideal for cohesionless soils only.

152 The Lade Model is limited to failure criteria for granular soils as well as normally consolidated clays. The
153 modified Cam clay model takes into account elastoplastic behaviour of soil leaving alone none-linear elasticity
154 characteristics prior to yielding. The Egg Camclay model addresses precisely the nonlinear elasticity and plasticity
155 of the soil. Of the failure criteria for clay soils in subcritical region, the Modified Egg Cam Clay is the most
156 appropriate one for the description of expansive soil behavior with reasonable accuracy. The model is superior
157 because it is characterized with the limited number of constitutive parameters easily determined in the laboratory
158 or even in situ. Indeed, engineers can make use of this model which provides a reasonable fit to data obtained
159 from laboratory tests.



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Figure 1: Figure 1 :Figure 2 :



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Figure 2: $2 J =$ second stress deviator invariant $1 I =$ first stress invariant $1 I \& 2 J$ Volume Figure 3 :Figure 4 :

5

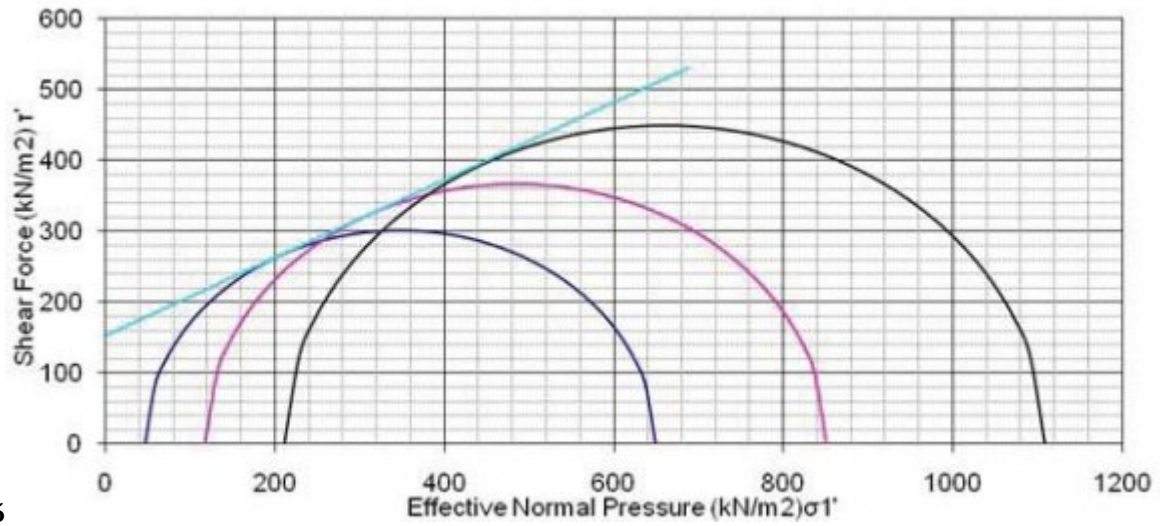


Figure 3: Figure 5 :

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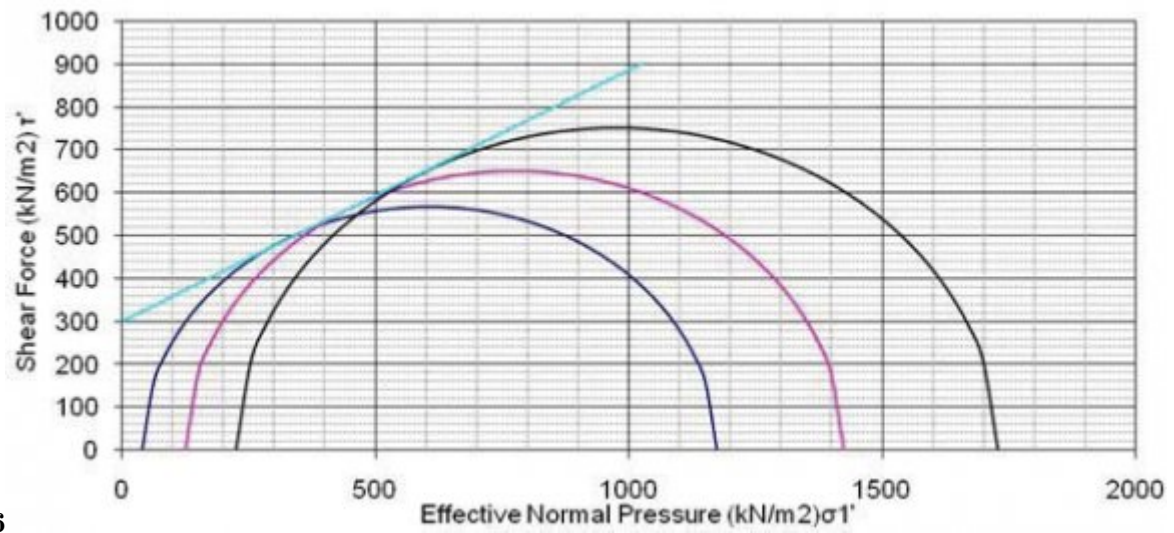
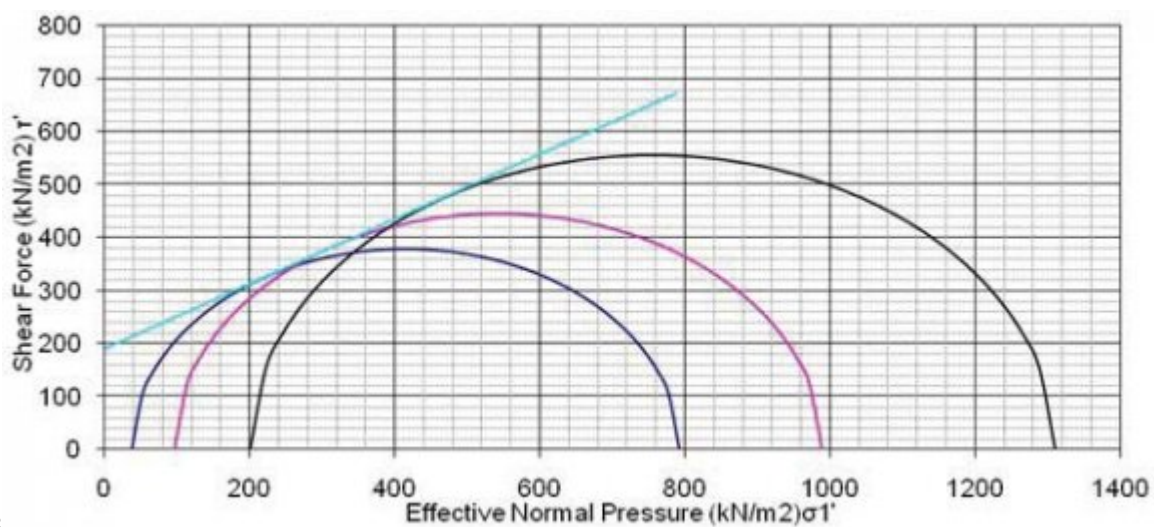


Figure 4: pFigure 6 :f

9 CONCLUDING REMARKS



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Figure 5: pCFigure 7 :Figure 8 :



913

Figure 6: Figure 9 : 1 I 3 I

1

Bulk density kg/m ³	Dry density kg/m ³	Density of solids kg/m ³	Swell potential %	Swell pressure kPa	Compaction (Heavy Proctor) MDD kg/m ³	UCS kN/m ²	Triaxial test (CU) c
2120	1910	2650	19.2	560	1944	11.7 106	14 17

Figure 7: Table 1 :

1

that the modified Cam Clay Model gives a very good

	Untr. Soil	4% Lime	6% Lime	8% Lime
$[\phi]$	14	31	32	33
c [kN/m ²]	17	152	300	187

Figure 8: Table 1 :

Figure 9:

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