

# Statistical Investigation of ECG Signal of Sleep Apnea Patient

K.M.Hossain<sup>1</sup>

<sup>1</sup> Department of Electronics and Instrumentation Engineering

*Received: 10 November 2011 Accepted: 2 December 2011 Published: 17 December 2011*

---

## Abstract

The Hurst Exponent of the time series of a normal patient and apneal patient suggest that they are anti-persistent and the later has more self similarity compared to the former. It has been established that they are AR process and nonstationary. The Semblance analysis suggests strong correlation both positive and negative between them. Tentative mathematical models of the normal an apneal patient has also been suggested using Yule Walker method.

---

*Index terms*— Hurst exponent, ECG, autocorrelation, partial autocorrelation, Wavelet, Semblance

## 1 Statistical Investigation of ECG Signal of Sleep Apnea Patient

Chandan Das<sup>?</sup>, Mofazzal H. Khondekar<sup>?</sup> A Abstract -The Hurst Exponent of the time series of a normal patient and apneal patient suggest that they are anti-persistent and the later has more self similarity compared to the former. It has been established that they are AR process and nonstationary. The Semblance analysis suggests strong correlation both positive and negative between them. Tentative mathematical models of the normal an apneal patient has also been suggested using Yule Walker method.

Keywords: Hurst exponent, ECG, autocorrelation, partial autocorrelation, Wavelet, Semblance.

leep apnea is the occurrences of interrupted breathing during sleep. Obstructive sleep apnea is a well-known disorder in which relaxation of muscles in the throat repeatedly close off the airway during sleep; the person wakes just enough to take a gasping breath. This process is repeated many times during sleep and usually is not remembered the next day. Those suffering from severe obstructive sleep apnea typically complain of sleepiness, irritability, forgetfulness, and difficulty in concentrating. They may have difficulties in their occupational or social lives and be prone to motor vehicle accidents. The disorder has been medically linked to hypertension, which in turn puts people at greater risk of heart failure and stroke.

An electrocardiogram (ECG or EKG, abbreviated from the German Elektrokardiogramm) is a graphic produced by an electrocardiograph, which records the electrical activity of the heart over time [1]. Its name is made of different parts: electro, because it is related to electronics, cardio, Greek for heart, gram, a Greek roots meaning "to write". Specific waveforms within the ECG represent the electrical activity associated with mechanical events such as ventricular contraction and relaxation (systole and diastole). Analysis of the various waves and normal vectors of depolarization and re-polarization yields important diagnostic information [2].

ECG signals of the normal patient and apnea patient being taken for a period of 15minutes [3, 4] with the sampling interval of 4 msec. In this paper we will try to find out the nature of variability of the above two ECG signals using Finite Variance Scaling Method (FVSM). But before we proceed for the above action we have to consider that in practical cases all the observed data involve some amount of circumstantial errors which may creep in due change in environment, or systematic error which is due to factors inherent in the manufacture of the measuring instrument arising out of tolerances in the components of the instruments. Study of such data in presence of error may often not succeed to give true information. There is the need to remove these errors up to a satisfactory level. For these purpose we frequently use different methods of filtration in the time-dependent data. Here Simple Exponential Smoothing technique has been used for the filtration purpose.

The Hurst Exponent obtained from FVSM quantifies the relative affinity of a time series either to regress strongly to the mean or to cluster in a direction. Autocorrelation plots are used for checking randomness in a data set. This randomness is estimated by computing autocorrelations for data values at varying time lags. For

46 random time series, such autocorrelations are near zero value for every time-lag, whereas for deterministic series,  
 47 one or more of the autocorrelations will have notably non-zero values.

48 Partial autocorrelation plots are used here for model identification in Box-Jenkins models of the time series.

49 Semblance Analysis using the continuous wavelet transform has been done to investigate the similarity of the  
 50 phase relationship locally between the two signals which is a function of frequency and time of the signals.

## 51 2 a) Simple Exponential Smoothing

52 Exponential Smoothing helps to produce a smoothed Time Series by assigning exponentially decreasing weights  
 53 as the observation in the time series get older. Simple Exponential Smoothing [5] where  $y_i$  is the smoothed data  
 54 at the  $i$ -th position and  $\alpha$  ( $0 < \alpha < 1$ ) is a parameter. This is equivalent to  $y_i = \alpha x_i + (1-\alpha)y_{i-1}$  and where the sum of the  
 55 corresponding weights  $\alpha, \alpha(1-\alpha), \alpha(1-\alpha)^2, \alpha(1-\alpha)^{i-2}$  and  $(1-\alpha)^{i-1}$  is equal to unity. Thus in effect, each  
 56 smoothed value is a convex linear combination of all the previous observations as well as the current observation.

57 A familiar version of Finite Variance Scaling Method (FVSM) is the Standard Deviation Analysis (SDA)  
 58 [6,7,8], which is based on the assessment of the standard deviation  $D(t)$  of the variable  $x(t)$ . In a time series  
 59  $\{x(t_i)\}$  observed at the instants  $t_i$  for  $i=1, 2, \dots, n$  it yields  $\ln D(t) = \ln \sigma^2 + H \ln t$  (1) For  $n=1, 2,$   
 60  $3, \dots, j$  Eventually it is observed [6, 7 and 8]  $\ln D(t) = H \ln t + D_0$

61 The exponent  $H$  is known as the Hurst exponent. It is evaluated from the gradient of the best fitted straight  
 62 line in the log-log plot of  $D(t)$  against  $t$ . The value of the Hurst exponent ranges between 0 and 1. A value of  
 63 0.5 indicates a true random walk (a Brownian time series). In a random walk there is no correlation between any  
 64 element and future element. A Hurst exponent value  $0 < H < 0.5$  will exist for a time series with anti-persistent  
 65 behavior (negative autocorrelation) [9]. If the Hurst exponent is  $0.5 < H < 1.0$ , the process will be a long memory  
 66 process. A Hurst exponent value in this range indicates persistent behavior (or, a positive autocorrelation).

67 Autocorrelation is a statistical method used for time series analysis. It refers to the correlation of a time series  
 68 with its own past and future values. The values of the autocorrelation coefficients serve two purposes. It can  
 69 detect non-randomness in a data set. If the values in the data set are not random, then autocorrelation can help  
 70 the analyst choose an appropriate time series model.

71 The set of autocorrelation coefficients arranged as a function of separation in time is the sample autocorrelation  
 72 function (acf). If  $x_i$  be signal of length  $N$  and  $\bar{x}$  be its overall mean i.e. The autocorrelation coefficient at lag  $k$   
 73 is given by:  $r_{xx}(k) = \frac{1}{N-k} \sum_{i=1}^{N-k} (x_i - \bar{x})(x_{i+k} - \bar{x})$

74 The plot of the autocorrelation coefficients as a function of lag is called the correlogram.

75 Positive autocorrelation signifies the persistent trend in the series where the system likes to remain in the same  
 76 state from one observation to the next. Whereas negative autocorrelation is distinguished by an inclination for  
 77 positive departures from the overall mean  $\bar{x}$  to follow a negative departure, and vice versa.

78 In order to find the connection between  $r_{xx}(k)$  and  $H$  is the autocorrelation function at lag  $k$ .

79 Partial autocorrelation is a commonly used tool for model identification. If the sample autocorrelation plot  
 80 indicates that an AR model may be appropriate, then the sample partial autocorrelation plot is examined in order  
 81 to identify the order. We look for the lag on the partial autocorrelation plot beyond which its values essentially  
 82 become zero, more specifically where the values of the coefficients are considerably less than a 95% confidence  
 83 level i.e.

## 84 3 6

85 Where  $\psi$  is the mother wavelet, and  $\psi^*$  is complex conjugate of  $\psi$ ,  $s$  allows the wavelet to be stretched to various scales  
 86 and  $u$  allows the wavelet to be translated to by various displacements. The CWT basically is the convolution of  
 87 the signal with scaled version of the mother wavelet. Here, the complex Morlet wavelet has been used, which is  
 88 defined as [11, ??2]  $\psi(u, s) = e^{-i\pi u^2 / 2s} e^{-i\pi u^2 / 2s}$  (in Eq. (??)). When the mother wavelet chosen here is complex and hence its real and  
 89 imaginary parts generate a Hilbert transform pair, to order to have orthogonality. Since the mother wavelet is  
 90 complex, the CWT will also be complex which has a phase at every time and scale. The cross-wavelet transform  
 91 [13, ??4] defined as: The Hurst exponent that we have obtained for both the normal and apneal patient are less  
 92 than 0.5 which suggest that the signals are having anti-persistent behavior i.e. there are trends of a decrement  
 93 in values followed by an increment and vice versa and it is more pronounced in case of the apneal patient.

## 94 4 CWT CWT CWT

95 The Fractal Dimension ( $D$ ) is related to the Hurst exponent by the equation of  $D=2-H$ . Hence the  $D$  for the  
 96 normal patient is 1.7221 and for the apneal patient it is 1.87. These values of  $D$  suggest that the Global (F) <sup>1</sup>  
 97 <sup>2</sup>

<sup>1</sup>© 2011 Global Journals Inc. (US)

<sup>2</sup>November © 2011 Global Journals Inc. (US)



Figure 1:

I.

Figure 2:

1122 I

Figure 3: 11 For 2 Fig. 2 :

455 INTRODUCTION

Figure 4: Fig. 4 : 5 Fig. 5 :



---

98 apneal patient signal has more self similarity than that of a normal patient.

99 From the auto-correlogram as shown in fig. ?? we find that the autocorrelation coefficients die down to zero  
100 more rapidly than that of the apneal patient. The autocorrelation coefficients for apneal patient seem not to die  
101 down to zero except for large values of the lag. It signifies that the apneal patient's time series has a stronger  
102 trend compared to that of a normal patient. The auto-correlogram also suggests that both the systems from  
103 which the signals originated are Autoregressive (Markov) process (AR). The tendency of the autocorrelation  
104 coefficients of the apneal patient not to die quickly as compared to those of the normal patient can be taken as  
105 an indication of stronger nonstationarity of the former signal with respect to the later.

106 From the partial auto-correlogram as in fig. ?? we can claim that normal signal is auto-regressive process of  
107 order 9 i.e. AR (9) but the patient signal is autoregressive process of order 4 i.e. AR (4). Using the Yule Walker  
108 Equation [10], the model of the two data series can be estimated as

109 [Sarkar et al.] , A Sarkar , P Barat , P Mukherjee , S .

110 [Hossain et al.] , K Hossain , N Dipendra , Koushik Ghosh , Ghosh . Scaling Analysis Of National Stock Exchange  
111 Index

112 [Winkler and Makridakis ()] , R L Winkler , S Makridakis . *Journal of the Royal Statistical Society, Series A*  
113 *(General)* 1983. 146 (2) p. 150.

114 [Scafetta and Grigolini ()] , N Scafetta , P Grigolini . *Phys. Rev. E* 2002. 66 p. 36130.

115 [American Heart Association Guidelines for Cardiopulmonary Resuscitation and Emergency Cardiovascular Care -Part 8: Stabiliz  
116 'American Heart Association Guidelines for Cardiopulmonary Resuscitation and Emergency Cardiovascular  
117 Care -Part 8: Stabilization of the Patient With Acute Coronary Syndromes'. IV-89 -IV-110. *Circulation*  
118 2005. 112.

119 [Bandyopadhyay ()] Bandyopadhyay . *Proceedings of National Conference on Nonlinear Systems and Dynamics*  
120 *(held at A.M.U., Aligarh during, (National Conference on Nonlinear Systems and Dynamics (held at A.M.U.,*  
121 *Aligarh during) February 24-26, 2005) 2005. p. 155.*

122 [Mallat ()] *Computational Signal Processing with Wavelets*, S Mallat . 1998. 1998. New York; Boston, MA:  
123 Birkha" user Boston Inc. p. 352. (A Wavelet Tour of Signal Processing)

124 [Alfaouri and Daqrouq ()] 'ECG Signal Denoising By Wavelet Transform Thresholding'. M Alfaouri , K Daqrouq  
125 . *American Journal of Applied Sciences* 1546-923. 2008. 5 (3) p. .

126 [Braunwald E. (ed.) ()] *Heart Disease: A Textbook of Cardiovascular Medicine, Fifth Edition*, Braunwald E.  
127 (ed.) 1997. Philadelphia: W.B. Saunders Co. p. 108.

128 [Mandelbrot ()] *The Fractal Geometry of Nature*, B B Mandelbrot . 1983. New York: Freeman.

129 [Box and Jenkins] *Time Series Analysis-Forecasting and Control*, G E P Box , G M Jenkins , ReinselG . Pearson  
130 Education Ltd. (3 rd Edition)